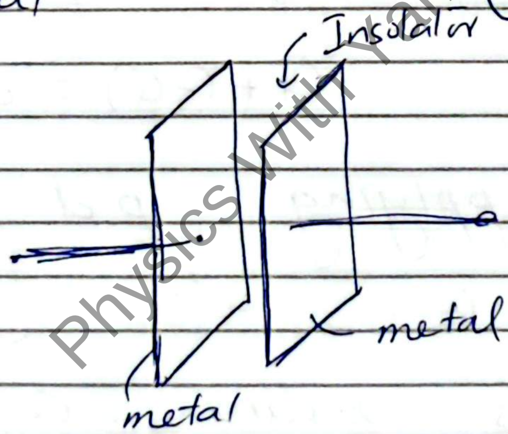
Capacitance

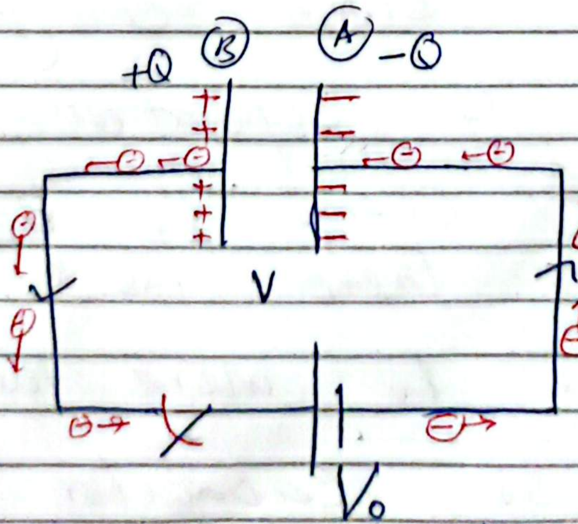
Capacitor:

A device/component that stores energy in the form of electric field.

Symbol

It is made up of two metal plates which are held parallel and they are seperated by insulating material





When switch is closed

electrons flow towards plate A. It becomes ⊖ charge

When switch is closed:

Electrons move away from plate B so it becomes ⊕ charge

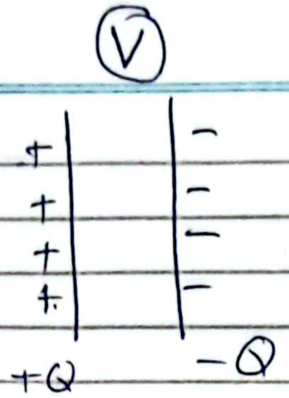
Flow of electrons will stop when potential difference between plates becomes equal to emf of battery.

total Charge on both plates = $Q + (-Q) = 0$

Before applying p.d ~~total~~ total charge was also zero

So this means capacitor does not store charge

But their is seperation of ⊕ and ⊖ charge so ~~B~~ this means ~~there~~ work has been done to move them apart so capacitor stores energy.



Due to charge on both plates there is a potential difference between them

$$Q \propto V$$

$$\boxed{Q = CV}$$

Where C is known as capacitance

Capacitance $C = \dfrac{Q}{V}$

Ratio of charge to potential difference accross capacitor.

Unit is Farad (F)

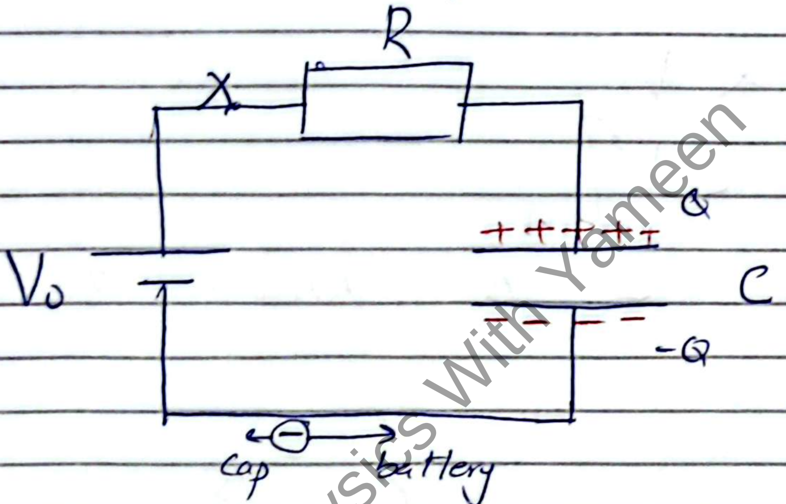
Define Farad (F)

$$F = \frac{\text{Coulomb}}{\text{Volt}} \qquad \boxed{C/V}$$

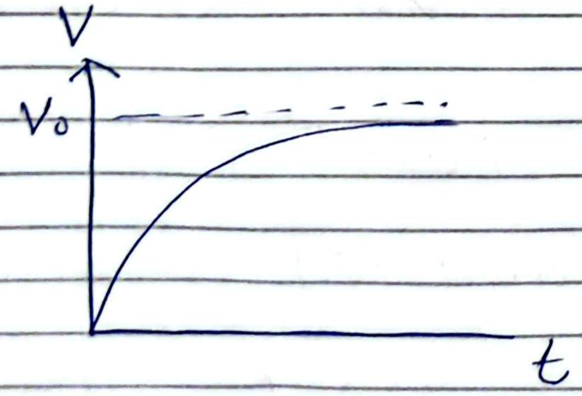
Micro farad $\rightarrow \mu F \rightarrow 10^{-6} F$

Nano farad $\rightarrow nF \rightarrow 10^{-9} F$

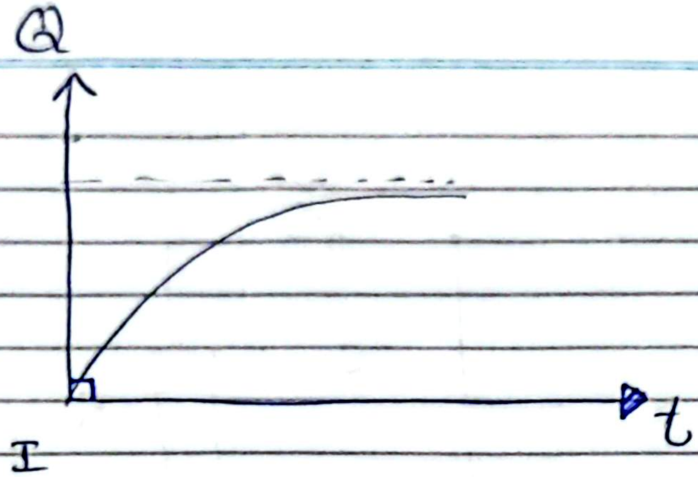
If we apply resistor in series to the circuit Capacitor charging time will be controlled



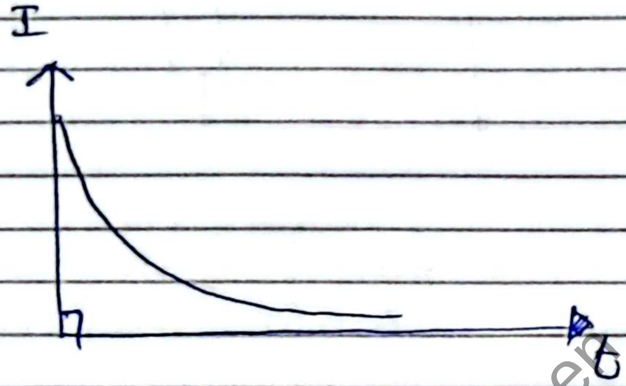
Voltage accross capacitor will increase with time



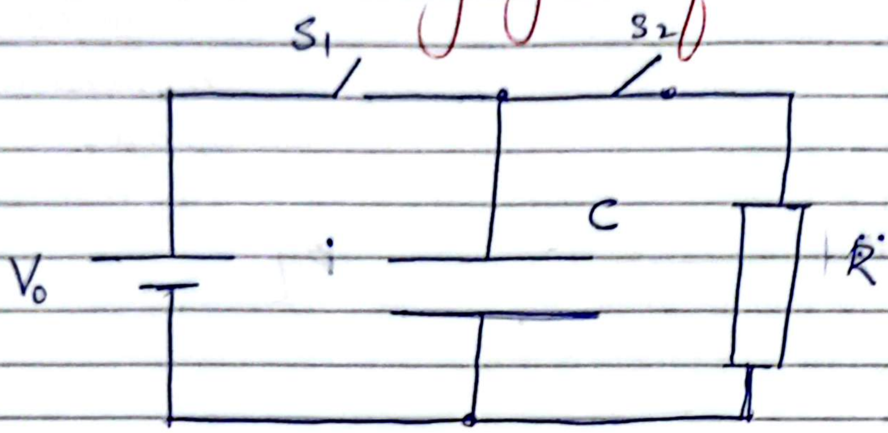
V: Voltage across the Capacitor



Rate of Charge will decrease with time.



Discharging of a capacitor



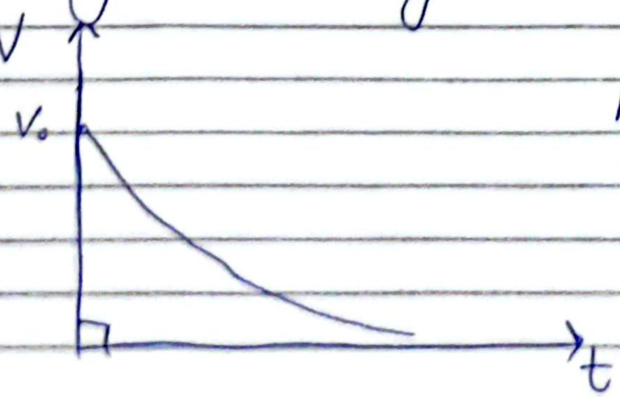
When S_1 is closed and S_2 is open the capacitor will charge upto V_0

When S_1 is opened and S_2 is closed the capacitor will discharge through R.

The voltage accross the capacitor will decrease as charge decrease

$$V = V_0 \, e^{-\frac{t}{RC}}$$

This is voltage accross capacitor during discharge at time



Rate of decrease of voltage is decreasing

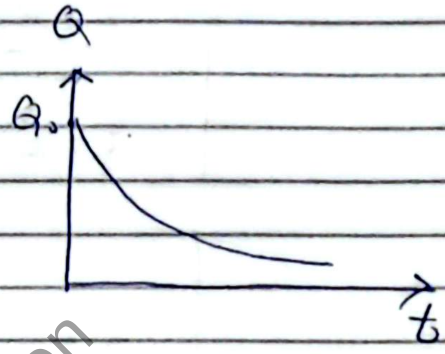
Time constant

product of R and C

$\therefore \tau = RC$ unit : second

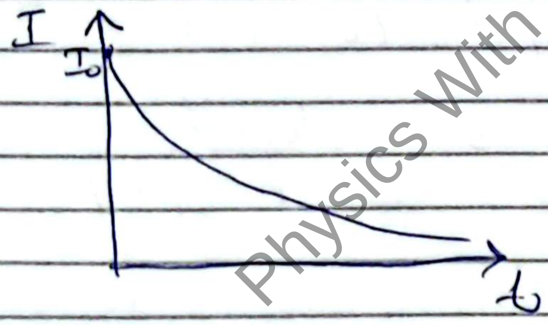
Equation of charge

$$Q = Q_0 e^{-\frac{t}{RC}}$$



current

$$I = I_0 e^{-\frac{t}{RC}}$$

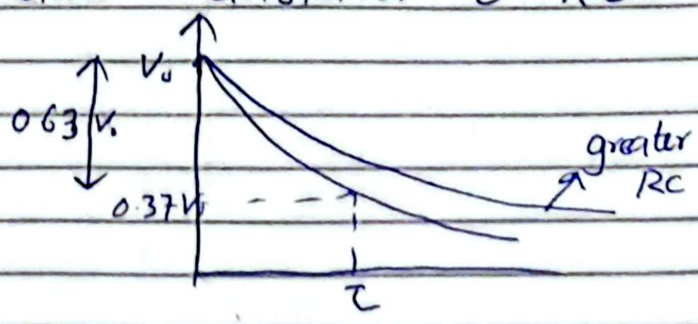


$$V = V_0 e^{-t/RC}$$

after one time constant $t = RC$

$$V = V_0 e^{-1}$$

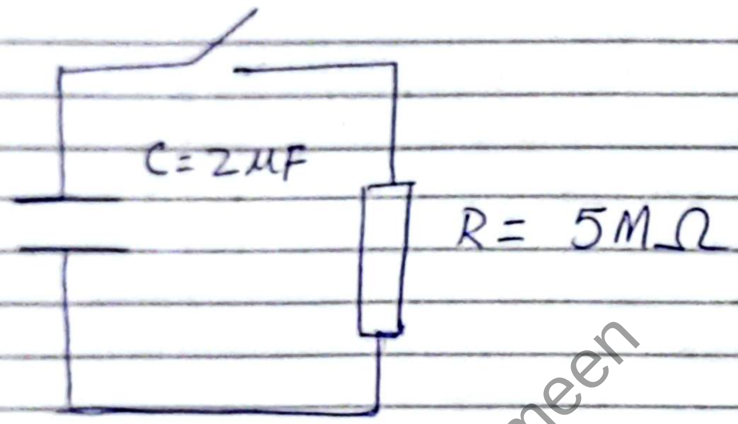
$$V = 0.37 V_0$$



In one time constant capacitor

discharges by 63 %

Example



Capacitor is charged to 10 V
when switch is closed
capacitor starts discharging

$$V = V_0 e^{-\frac{t}{RC}}$$

$V_0 = 10V$

$$V = 10 e^{-\frac{t}{5\times10^6 \times 2\times10^{-6}}}$$

convert
R into Ω
C " F

$$V = 10 e^{-\frac{t}{10}}$$

Find voltage at these time

t	V
0	10
1	9.0
2	8.2
3	7.4
⋮	
10	3.7
⋮	
50	0.1

$V = 10\, e^{-\frac{t}{10}}$

$V = 10\, e^{-\frac{1}{10}}$

Find time at which voltage accross the capacitor is 6V

$$V = V_0\, e^{-t/RC}$$

$$V = 10\, e^{-\frac{t}{10}}$$

$$6 = 10\, e^{-\frac{t}{10}}$$

$$\frac{6}{10} = e^{-\frac{t}{10}}$$

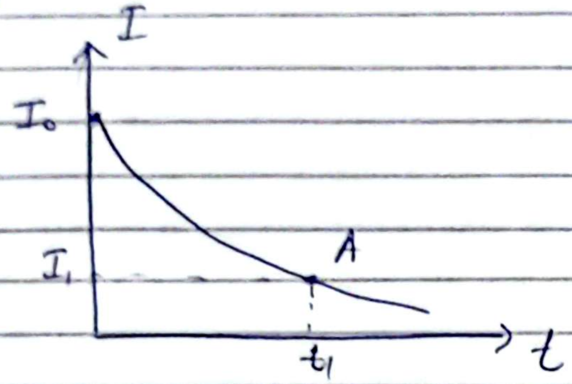
take ln on both sides

$$\ln\left(\frac{6}{10}\right) = \ln\left(e^{-\frac{t}{10}}\right)$$

$$\ln\left(\frac{6}{10}\right) = -\frac{t}{10} \ln e$$

$$t = 5.1\, s$$

Finding time constant from graph : Same for (V→t) graph
(Q − t) "
(I − t) "



$$I = I_0 e^{-\frac{t}{\tau}}$$ $RC = \tau$

take any point from the graph
lets take (A)

$$I_1 = I_0 e^{-\frac{t_1}{\tau}} \longrightarrow (1)$$

I_0 = known
I_1 = "
t_1 = "

Unknown τ

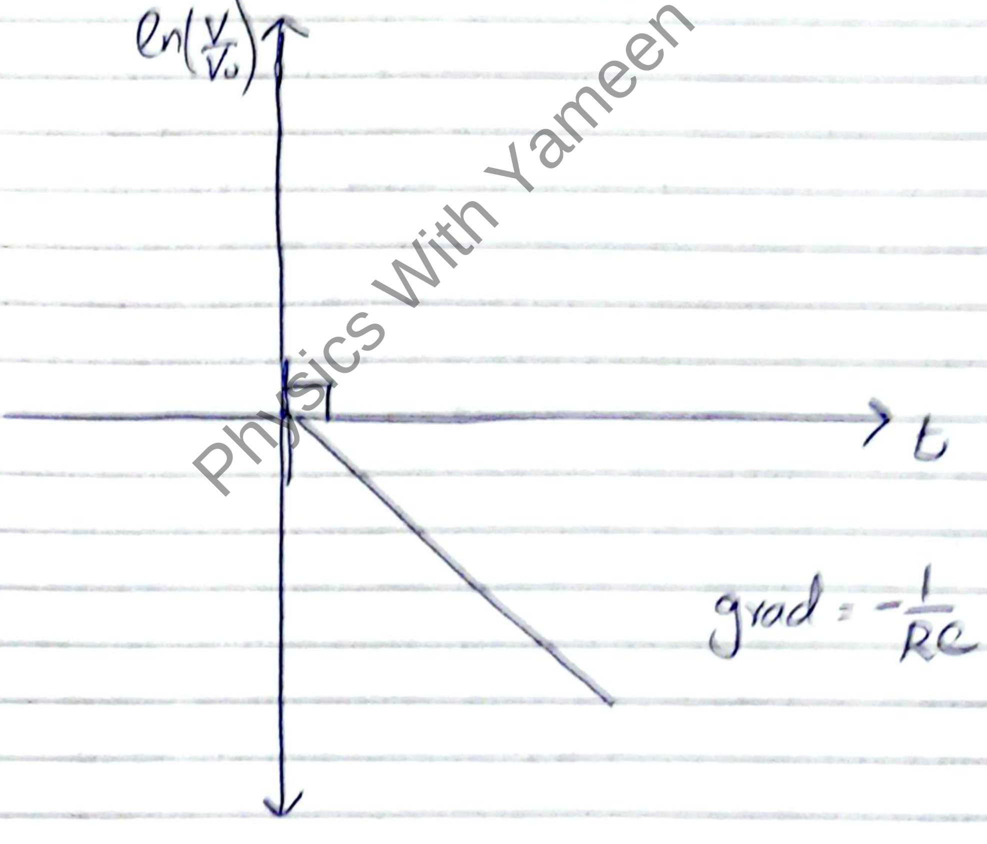
solve eq (1) to get τ

$$V = V_0 e^{-\frac{t}{RC}}$$

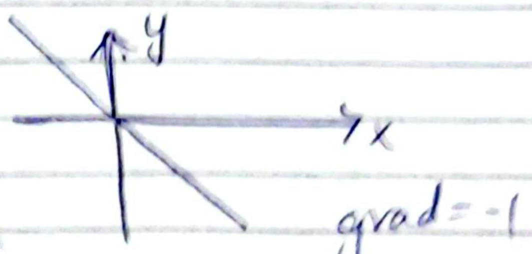
$$\frac{V}{V_0} = e^{-\frac{t}{RC}}$$

$$\ln\left(\frac{V}{V_0}\right) = \ln\left(e^{-\frac{t}{RC}}\right)$$

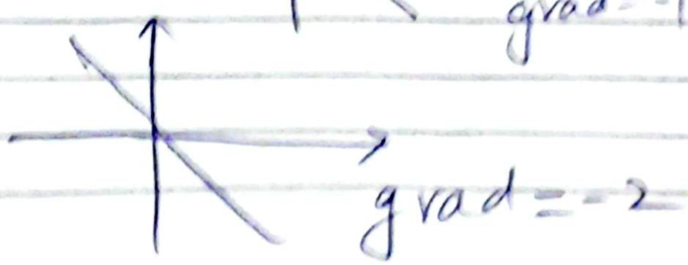
$$\ln\left(\frac{V}{V_0}\right) = -\frac{t}{RC}\,\overset{1}{\cancel{\ln e}}$$



$y = -x$



$y = -2x$

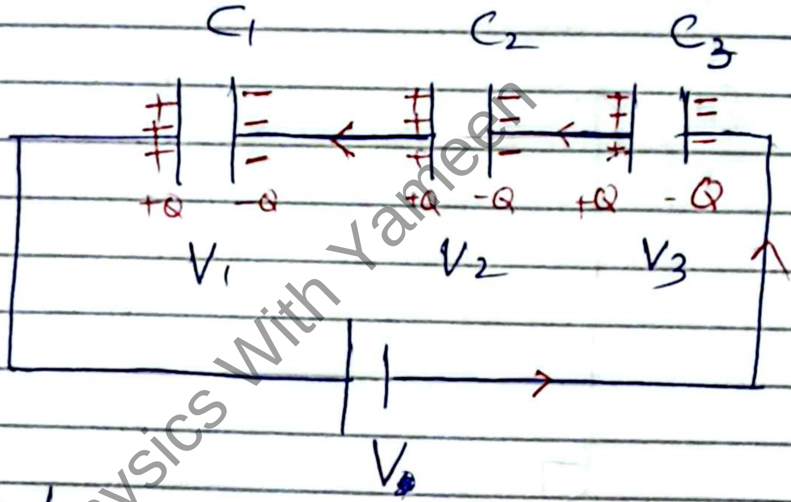


Combinations of capacitors

① Series

② parallel

<u>Series</u>



properties

× charge accross each capacitor is same

× Voltage accross each is different $(Q = CV)$

$$V = \frac{Q}{C}$$

Q is same $\quad V \propto \frac{1}{C}$

as capacitors are different so V is different

$$V = V_1 + V_2 + V_3$$

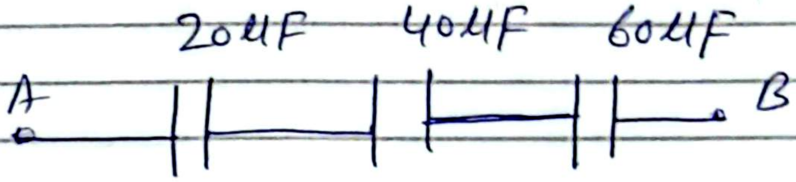
$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\boxed{\frac{1}{C_{tot}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

For two capacitors in series

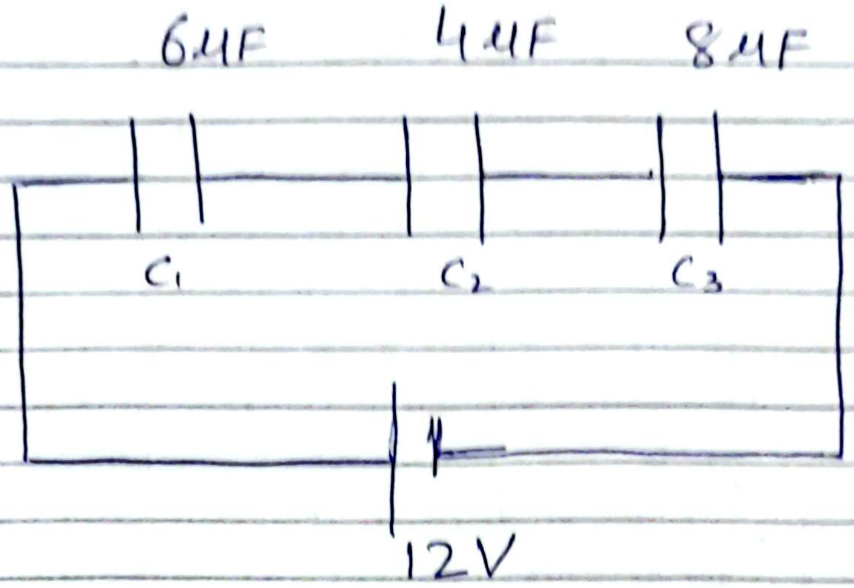
$$C_{tot} = \frac{C_1 C_2}{C_1 + C_2} \qquad \frac{\text{Product}}{\text{Sum}}$$



$$\frac{1}{C_{tot}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_{tot}} = \frac{1}{20} + \frac{1}{40} + \frac{1}{60}$$

$$C_{tot} = 10.9\ \mu F$$



i) Find total capacitance

ii) Find total charge transferred by the battery

iii) Find charge accross each capacitor

iv) Find potential difference accross potential difference.

$$\frac{1}{C_{tol}} = \frac{1}{6} + \frac{1}{4} + \frac{1}{8}$$

$C_{tol} = 1.85\ \mu F$ or $1.85 \times 10^{-6} F$

$Q_{tot} = C_{tot} V$

$Q_{tot} = 1.85 \times 12 = 22.2\ \mu C$

Charge accross each cap is same as total charge.

$Q = 22.2\,\mu C$

Voltage accross each

$V_1 = \frac{Q}{C} = \frac{22.2\,\mu C}{6\,\mu F}$

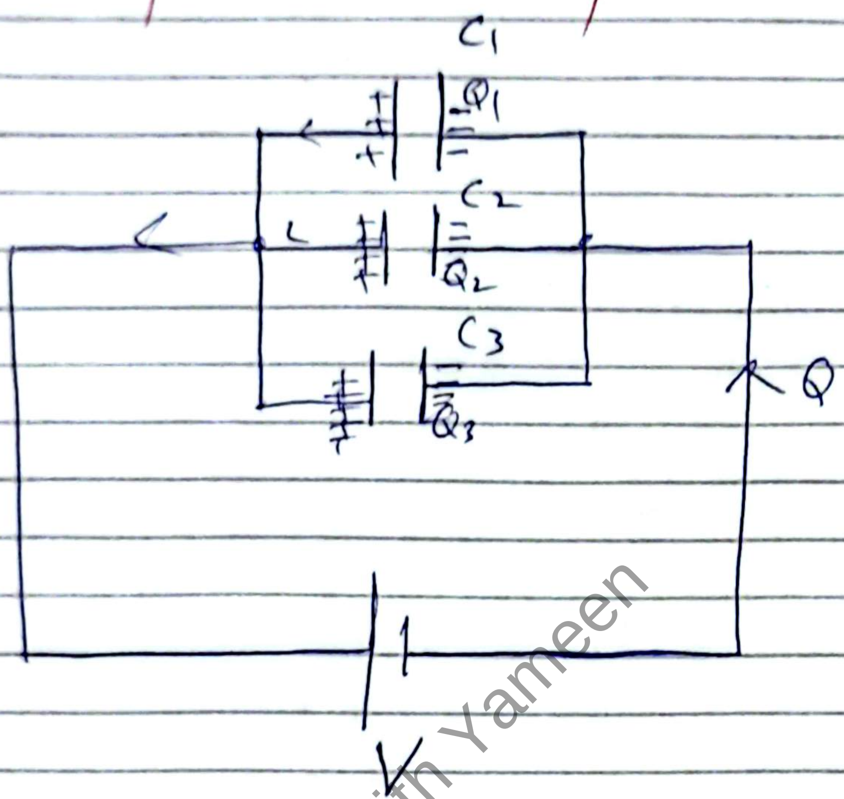
$V_1 = \frac{22.2 \times 10^{-6}\,C}{6 \times 10^{-6}\,F}$

$V_1 = 3.7\,V$

$V_2 = \frac{Q}{C_2} = \frac{22.2}{4}$

$V_3 = \frac{Q}{C_3} = \frac{22.2}{8}$

Capacitors in parallel.



properties

* p.d accross each is same (V)

* $Q = CV$ Charge accross each is different as V is same but c is differen

$Q \propto C$

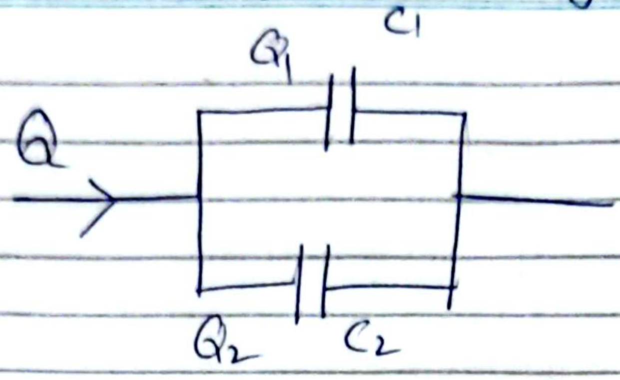
$$Q = Q_1 + Q_2 + Q_3$$

$$Q = C_1V + C_2V + C_3V$$

$$\frac{Q}{V} = C_1 + C_2 + C_3$$

$$\boxed{C_{tot} = C_1 + C_2 + C_3}$$

Charge dividing

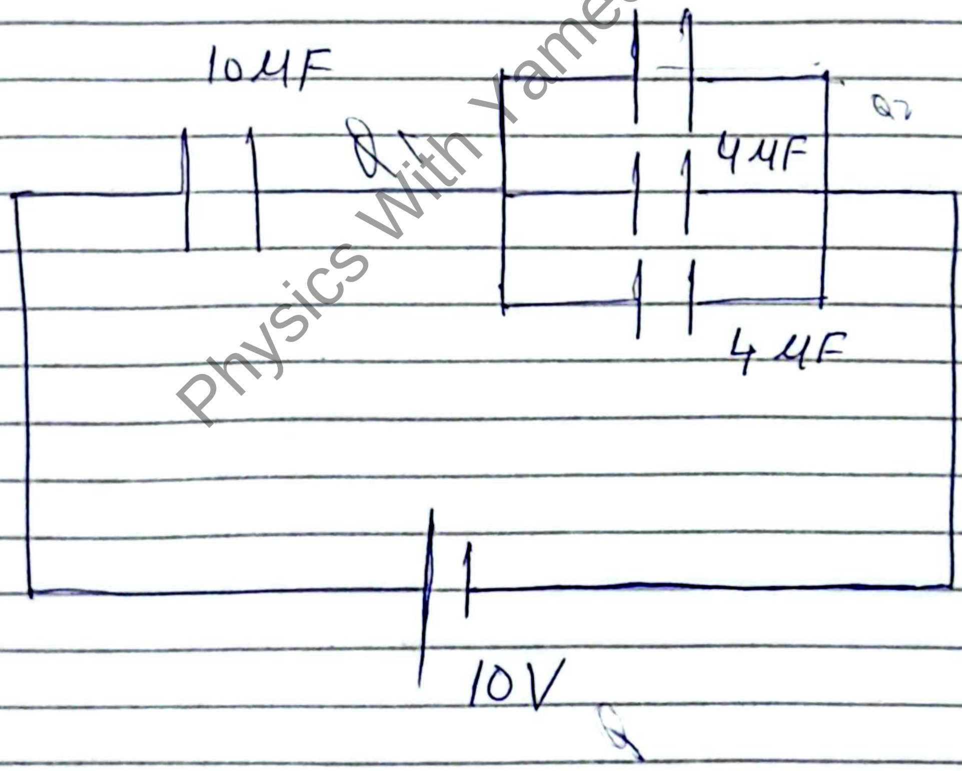


V: Same

$Q \propto C$

$$Q_1 = \frac{C_1}{C_1 + C_2} \times Q$$

Q: total



Find

i) total capacitance

ii) total charge

iii) charge accross each cap

(v) Voltage accross each capacitor

2, 4, 4, are in parallel

Case

$$\frac{1}{C_{tot}} = \frac{1}{10} + \frac{1}{10}$$

$$\frac{1}{C_{tot}} = \frac{2}{10}$$

$$\boxed{C_{tot} = 5\,\mu F}$$

$$Q = C_{tot} \times V$$

$$= 5 \times 10$$

$$Q = 50\,\mu C$$

Cap $10\,\mu F$ will have $50\,\mu C$ charge

but charge accross 2, 4, 4 will distribute

$$V_{10\mu F} = \frac{Q}{C}$$

$$= \frac{50\,\mu C}{10\,\mu F} = 5V$$

$$Q_{2\mu F} = 5 \times 2 = 10\,\mu C$$

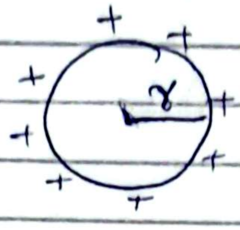
$$Q_{4\mu F} = 4 \times 5 = 20\,\mu C$$

$$Q_{4\mu F} = 4 \times 5 = 20\,\mu C$$

Charged Spherical conductors as capacitor.

Example: positively charged sphere

Q



We assume the Charge Q is at the center

β Electric potential at surface ?

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} \longrightarrow$$

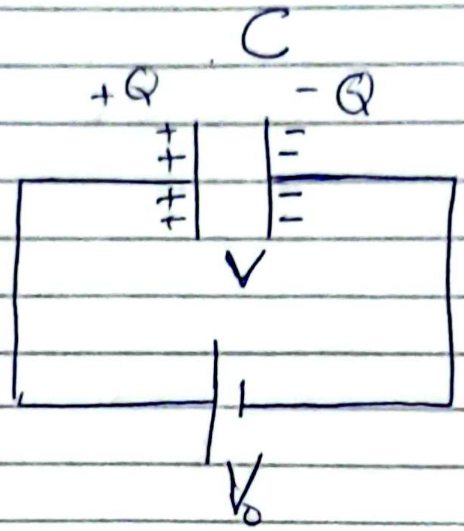
$$Q = CV$$

$$\frac{Q}{V} = C$$

$$4\pi\varepsilon_0 r = \frac{Q}{V}$$

$$\boxed{4\pi\varepsilon_0 r = C}$$

Energy stored in a capacitor.



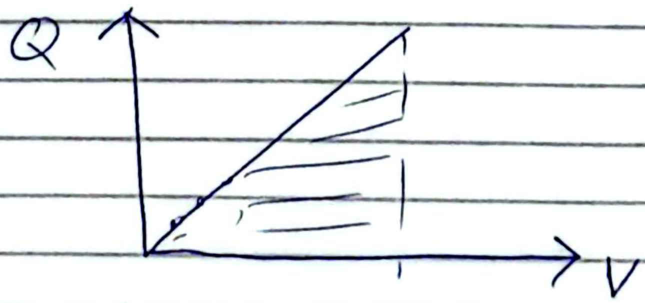
Charge on each plate is equal in magnitude but opposite in sign

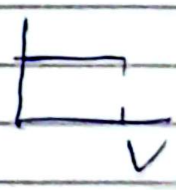
battery does work to separate ⊕ and ⊖ charges. This work done is stored in the form of Electric potential energy

$Q = CV$

$V = \frac{W}{Q}$

$W = VQ$





Area is equal to work done $= \frac{1}{2} QV$

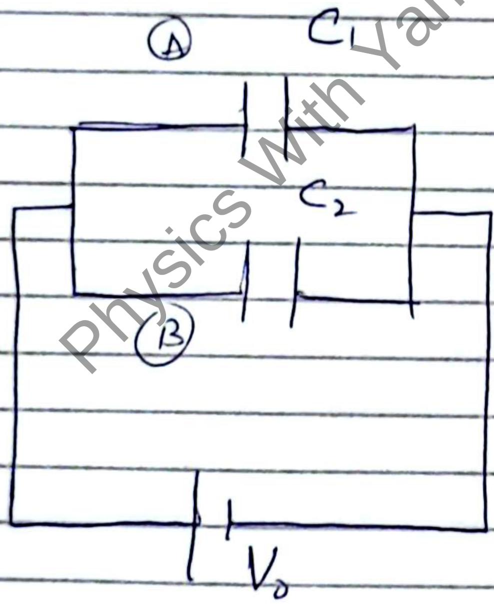
$$\text{Energy stored} = \frac{1}{2} QV \rightarrow (1)$$

We know $Q = CV$

put in (1)

$$\text{Energy} = \frac{1}{2} CV \cdot V$$

$$\boxed{\text{Energy} = \frac{1}{2} CV^2}$$



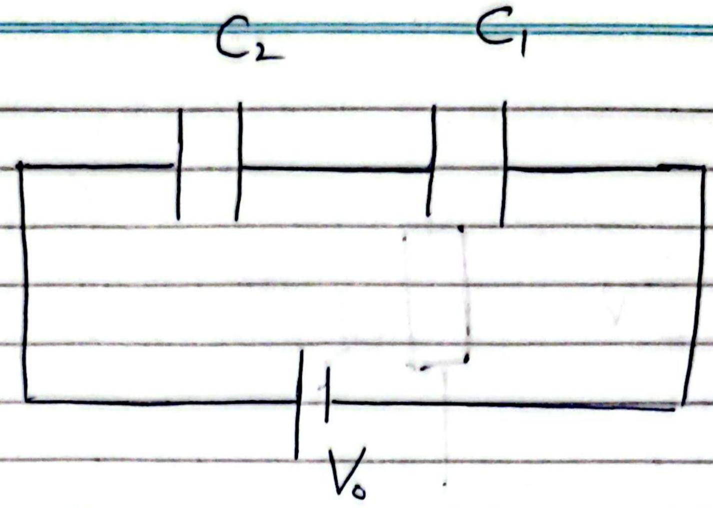
$E \propto C$
in this case

as V is same

Energy stored in A : $\frac{1}{2} C_1 V_0^2$

" " " B : $\frac{1}{2} C_2 V_0^2$

Ratio $$\boxed{\frac{E_A}{E_B} = \frac{C_1}{C_2}}$$



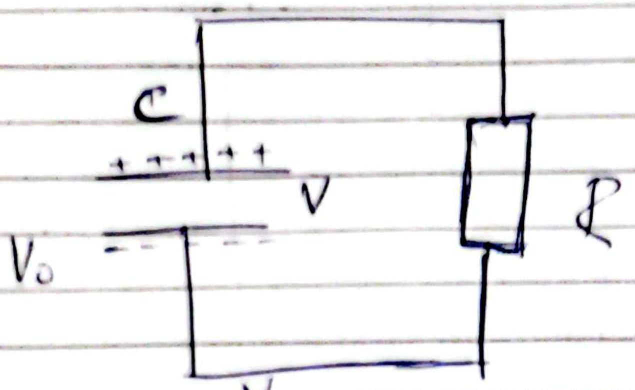
In series charge is same.
but voltage is different

$$V \propto \frac{1}{C}$$

$$E = \frac{1}{2} QV \qquad Q : \text{same}$$

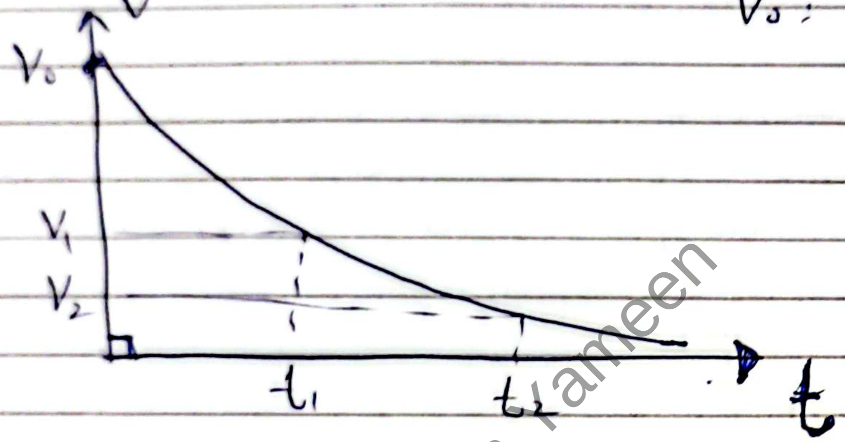
$$E \propto \frac{1}{C}$$

$$\boxed{\frac{E_{C_1}}{E_{C_2}} = \frac{C_2}{C_1}}$$



C : discharging

V_0 : Initial V_0



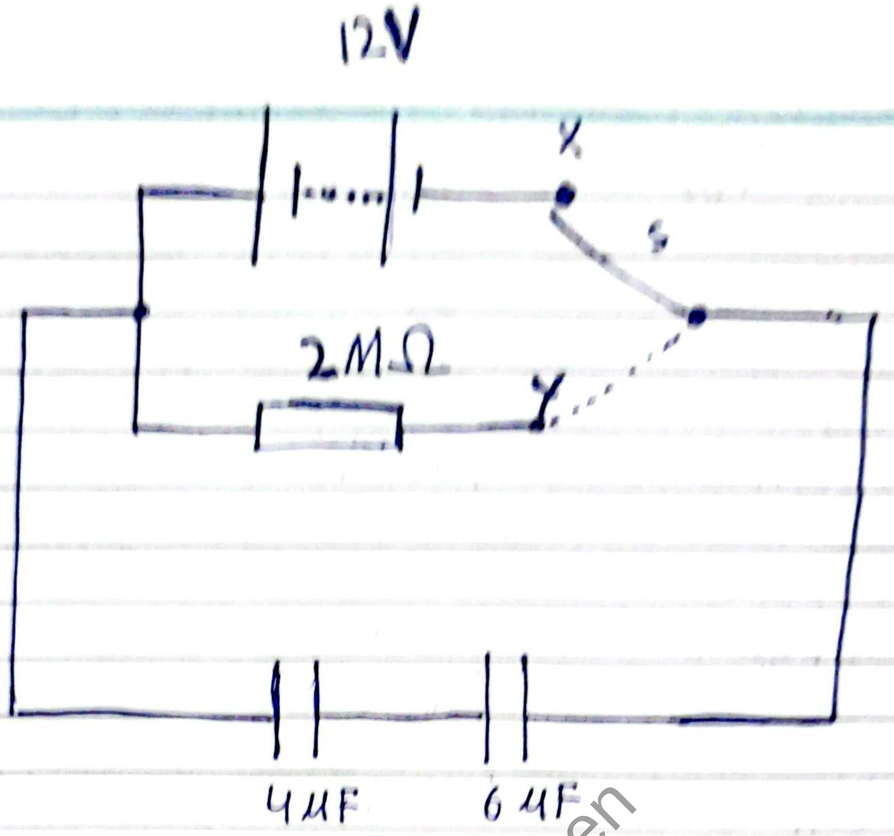
Find Loss of energy from t_1 to t_2 ?

at t_1 : $E_1 = \frac{1}{2} C V_1^2$

at t_2 $\quad E_2 = \frac{1}{2} C V_2^2$

Loss of energy $= \frac{1}{2} C V_1^2 - \frac{1}{2} C V_2^2$

" $\quad$ " $= \frac{1}{2} C (V_1^2 - V_2^2)$



when switch is at X both capacitor will charge

i) find voltage accross each capacitor.

$$C_{tot} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$$

$$\frac{1}{C_{tot}} = \frac{1}{4} + \frac{1}{6}$$

$$\frac{1}{C_{tot}} = \frac{3+2}{12}$$

$$C_{tot} = \frac{12}{5} = 2.4\ \mu F$$

$$Q_{tot} = C_{tot} \times 12 = 2.4 \times 12 = 28.8\ \mu C$$

$$V_{4F} = \frac{Q}{C}$$

$$= \frac{28.8}{4}$$

$$V_{4F} = 7.2\,V$$

$$V_{6\mu F} = \frac{28.8}{6}$$

$$V_{6\mu F} = 4.8\,V$$

When switch is connected is at y then capacitors will discharge.

Voltage accross both cap will decrease

$$V = V_0\, e^{-\frac{t}{RC_{tot}}}$$

$$V = 12\, e^{-\frac{t}{2\times10^{+6} \times 2.4\times10^{-6}}}$$

$$V_{tot} = 12 \cdot e^{-\frac{t}{4.8}}$$

find total voltage at

$t = 1\,s$, $t = 10\,s$, $t = 15\,s$

$V = 12\,e^{\frac{-1}{4.8}}$ $V = 12\,e^{\frac{-10}{4.8}}$

$V = 9.74\,V$ $V = 1.49\,V$

at $t = 15\,s$

$V = 12\,e^{-\frac{15}{4.8}}$

$V = 0.527\,V$

If we have to find Voltage accross any capacitor

find voltage accross $\underline{\underline{4\mu F}}$ at $t = 2\,s$

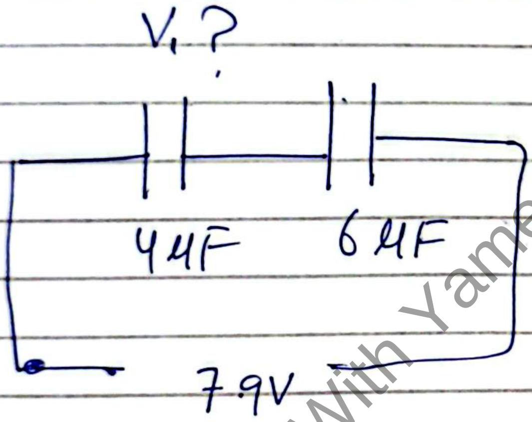
There are Two methods

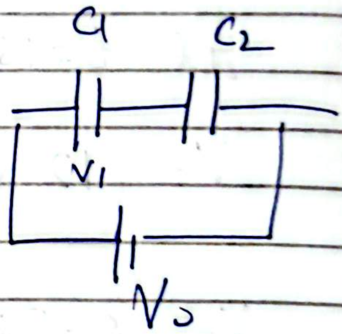
① Find total voltage after $2\,s$

$$V = V_0 \, e^{-t/R_{tot} C_{tot}}$$

$$V = 12 \, e^{-\frac{2}{4.8}}$$

$$V = 7.9\,V$$





$$V_1 = \frac{C_2}{C_1 + C_2} \times V_0$$

$$V_1 = \frac{6}{6+4} \times 7.9$$

$$\boxed{V_1 = 4.74\,V}$$

2nd Method

find initial voltage accross 4 μF :

$$V_{4\mu F} = 7.2\,V$$

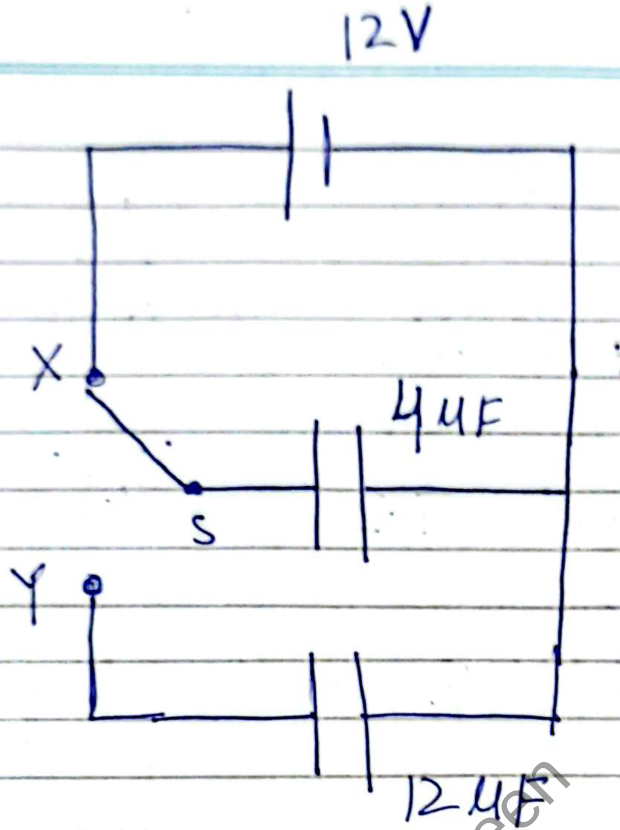
Use This

$$V_{4\mu F} = V_{0\,4\mu F}\; e^{-\frac{t}{R\,C_{tot}}}$$

$$V_{4\mu F} = 7.2\; e^{-\frac{2}{4.8}}$$

$$V_{4F} = 4.74.$$

Same as we got by 2nd method



When switch is at X

4μF will get charge

$Q = CV$

$Q = 4 \times 12 = 48 \mu C$

when switch is connected at Y 4μF will discharge and 12μF will charge

This processe will stop when Potential differenc

Final Charge accross each will be different.

accross both will be same.

If we have to find the final potential difference?

$$Q = Q_{4\mu F} + Q_{12\mu F}$$

$$Q = C_{4\mu F} V + C_{12\mu F} V$$

$$48\,\mu F = 4_{\mu F} V + 12_{\mu F} V$$

$$48\,\mu F = 16_{\mu F} V$$

$$48 \times \cancel{10^{-6}} = 16 \times \cancel{10^{-6}} V$$

$$\boxed{3\,V = V}$$