

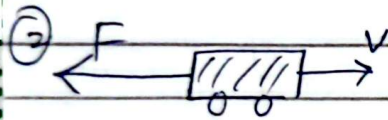
Motion in a circle

Circular Motion

"Motion of an object around circular path"

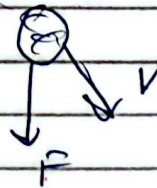
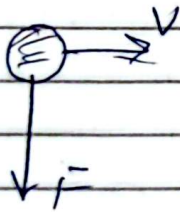


Speed will increase
but direction will
remain same



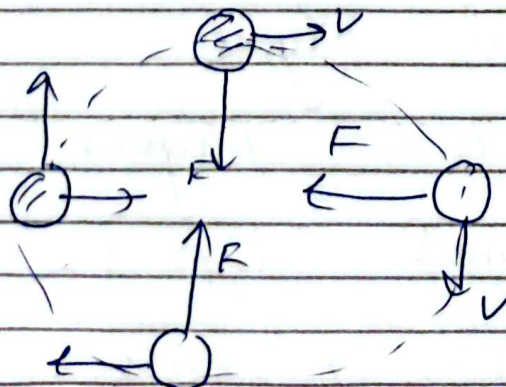
Slow down but
Direction remain same

③



projectile motion

④

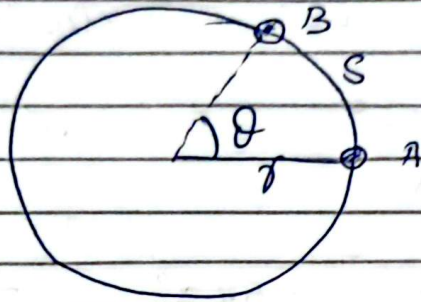


force and
velocity are

perpendicular
to each other

Basic terms

Angular displacement



If the object moves from point A to point B. It subtends angle θ at the center.

Angle subtended at the center of circle by an object having circular motion.

Unit: Degree and radian

Define 1 radian:

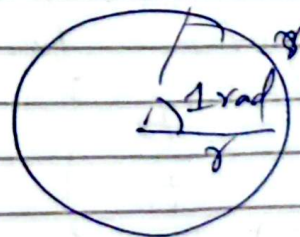
"[Angle subtended at the center of circle by an arc of length equal to radius of circle] is known as one radian]

$$S = r\theta$$

$$\theta = \frac{S}{r}$$

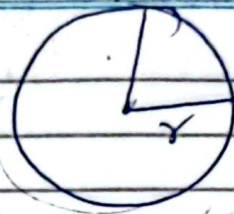
$$\text{if } S = r$$

$$\theta = 1 \text{ rad}$$



$$C = 2\pi r$$

total angle = 2π rad



$\frac{s}{r}$

$$2\pi \text{ rad} = 360^\circ$$

arc length angle

$$1 \text{ rad} = \frac{360^\circ}{2\pi}$$

$$\frac{r}{2\pi r} \times \frac{1}{\pi}$$

$$1 \text{ rad} = \frac{180^\circ}{\pi} \approx 57.3^\circ$$

$$\cancel{2\pi} = 2\pi \cancel{\pi}$$

$$\frac{1}{3} \text{ rad} = \text{--- degree}$$

$$\frac{1}{3} \times \frac{180}{\cancel{\pi}} = 60^\circ$$

$$360^\circ = 2\pi \text{ rad}$$

$$1^\circ = \frac{2\pi}{360} \text{ rad}$$

$$1^\circ = \frac{1}{180} \text{ rad}$$

Angular velocity (ω) $\rightarrow$ symbol

"Rate of change of angular displacement"

$$\omega = \frac{\theta}{t}$$

Formular

Units:

- ① radian per second
- ② Revolution per minute (RPM)

From rad/s to RPM

OR RPM to rad/s

120 RPM $\rightarrow$ ---- rad/s

$$\frac{120 \text{ R}}{\text{M}} \rightarrow \frac{120 \times 2\pi}{60 \text{ s}} \text{ rad}$$

X with $\frac{\pi}{30}$ $\frac{120 \times \pi}{30}$ rad/s

~~40~~ 4π rad/s

From rad/s $\rightarrow$ RPM

$$\frac{4\pi \text{ rad}}{\text{s}} \Rightarrow \frac{4\pi \times 60}{2\pi} \text{ RPM}$$

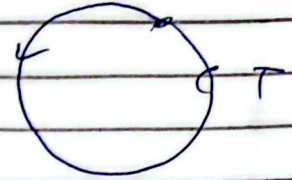
$$4\pi \text{ rad/s} \Rightarrow 120 \text{ RPM}$$



Relationship of angular velocity
and time period (T).

$$\omega = \frac{\theta}{t}$$

$$\omega = \frac{2\pi}{T}$$



Earth rotated about its own

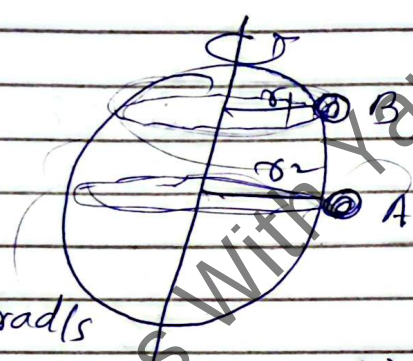
axis

$$T = 24 \text{ hrs}$$

$$\omega = \frac{2\pi}{24 \times 3600} \text{ rad/s}$$

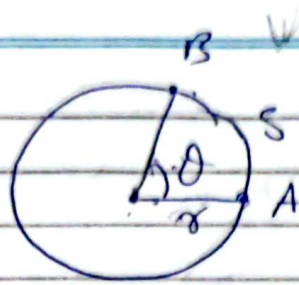
$$\omega = 7.27 \times 10^{-5} \text{ rad/s}$$

There are two point A and B



Both points have same angular
velocity as of earth

⇒ Relation b/w Linear speed
and angular speed

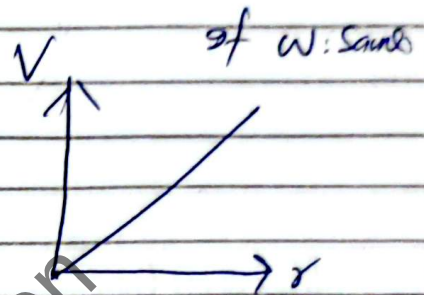


$$s = r \theta$$

Divide by time

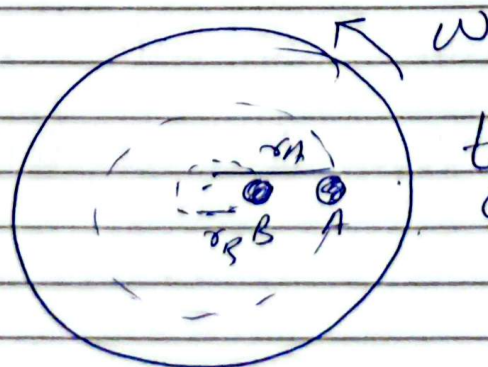
$$\frac{s}{t} = \frac{r \theta}{t}$$

$$v = r \omega$$



In previous Example A and B have same angular velocity but they have different linear speed

Linear speed of A > Linear speed of B



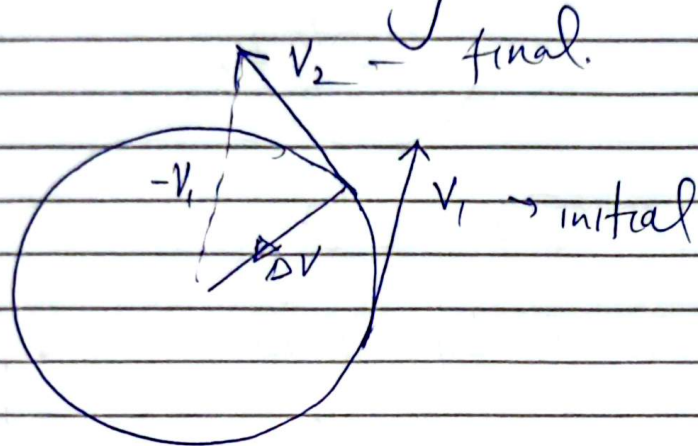
turntable or disc

Both A and B have same ω

$$v_A > v_B$$

$$r_A > r_B$$

Change in velocity

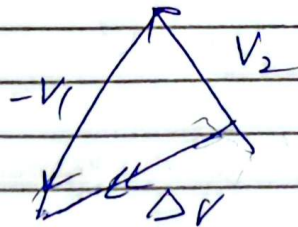


Magnitudes of v_1 and v_2 are same (uniform circular motion)

Δv = change in velocity

$$= v_2 - v_1$$

$$= v_2 + (-v_1)$$



In uniform circular motion

Δv is towards center

$$a = \frac{\Delta v}{\Delta t}$$

So a is towards center

Acceleration is perpendicular to velocity this is called as centripetal acceleration

Centripetal acceleration

acceleration that is perpendicular to velocity

$$a_c = \frac{v^2}{r}$$

OR

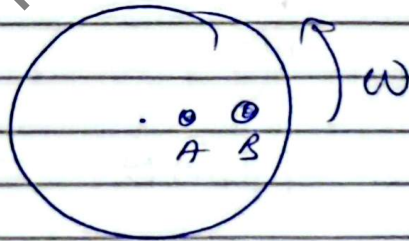
$$a_c = \frac{v^2}{r} = \frac{r\omega^2}{r} = r\omega^2$$

put $v = r\omega$

$$a_c = \frac{r^2\omega^2}{r}$$

$$a_c = r\omega^2$$

$$a_c(B) > a_c(A)$$



Centripetal force

In uniform circular motion net force is always lowards the center i.e. it is perpendicular to velocity

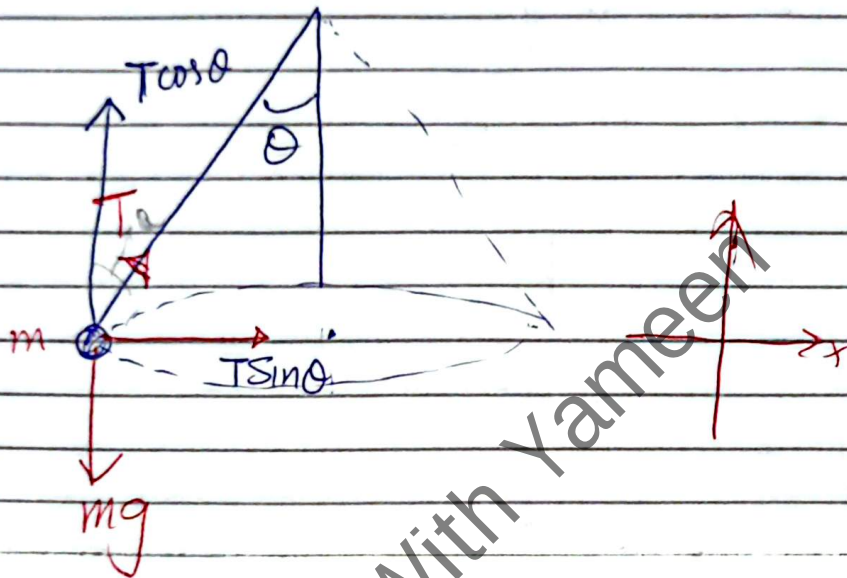
$$F_c = m a_c$$
$$F = \frac{mv^2}{r}$$

$$F = m r \omega^2$$

Applications of Circular Motion

①

Horizontal circular Motion



$$T \sin \theta = \frac{mv^2}{r} \rightarrow (1)$$

$$T \cos \theta = mg \rightarrow (2)$$

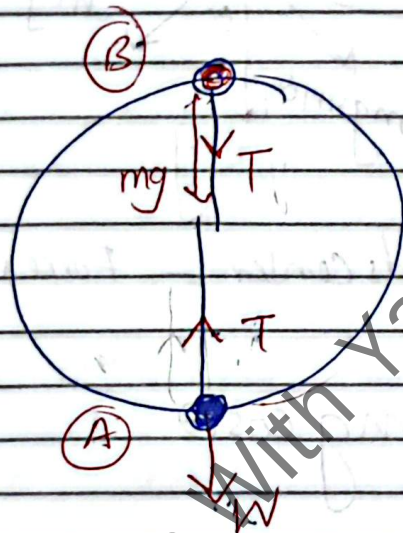
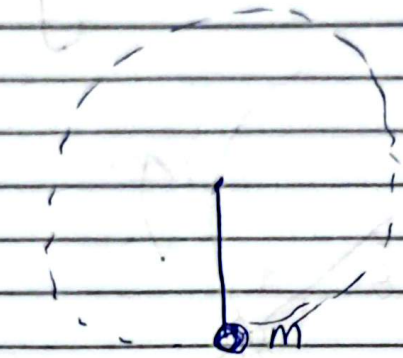
Divide Equation ① by ②

$$\frac{T \sin \theta}{T \cos \theta} = \frac{mv^2/r}{mg}$$

$$\boxed{\tan \theta = \frac{v^2}{rg}}$$

if $v \uparrow$ then $\theta \uparrow$

② Vertical circular motion



Speed is kept constant

At point A

$$T - W = \frac{mv^2}{r}$$

$$T_A = \frac{mv^2}{r} + mg$$

$$T_A > mg$$

at B

$$mg + T = \frac{mv^2}{r}$$

$$T_B = \frac{mv^2}{r} + mg$$

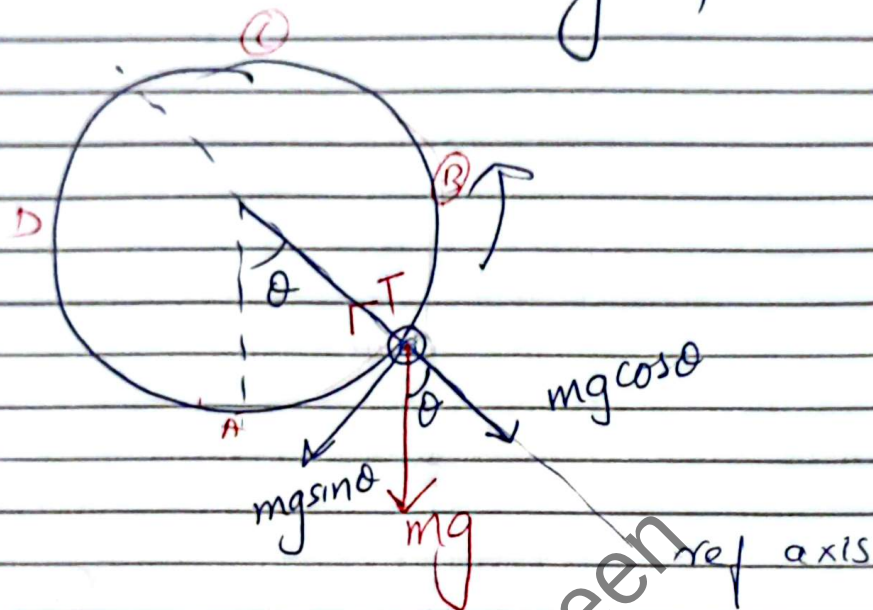
$$T_B < T_A$$



$$y = 2 + \cos x$$



Tension at any position

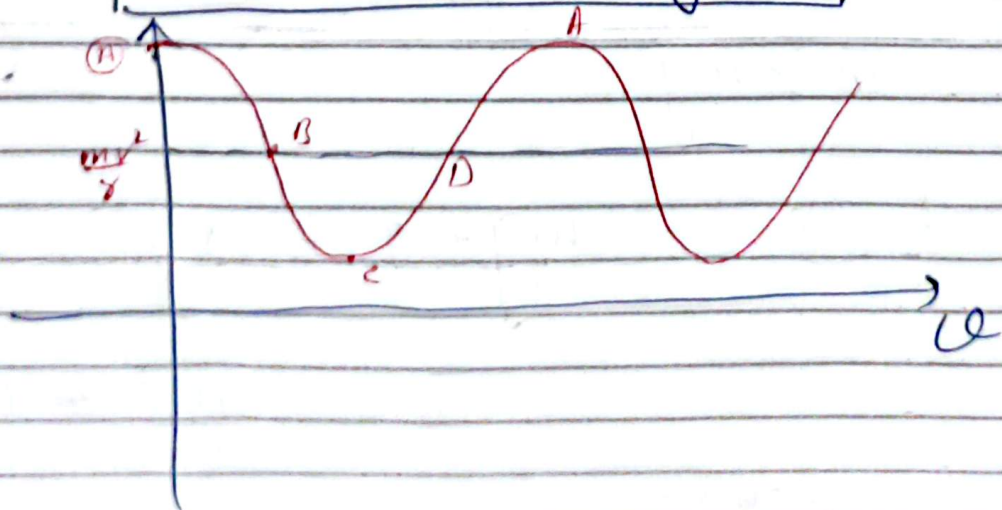


F_c = Forces towards center - Forces away from center

$$F_c = T - mg \cos \theta$$

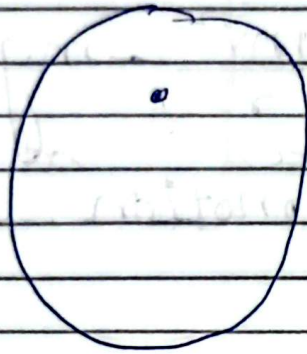
$$\frac{mv^2}{r} = T - mg \cos \theta$$

$$T = \frac{mv^2}{r} + mg \cos \theta$$



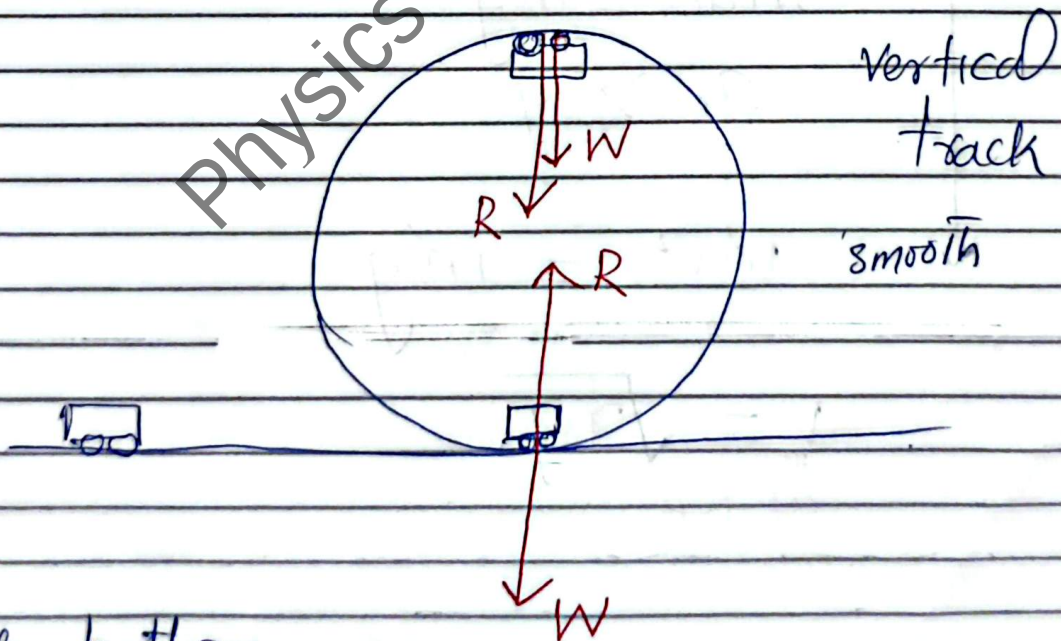
of Elastic string is used
then Draw path

$$F = kx$$



As tension
Changes so
extension
Changes

extension at
bottom is maximum and
extension at top is minimum
so the path is oval.



at bottom

$$R - W = \frac{mv^2}{r}$$

$$R = \frac{mv^2}{r} + mg$$

at top

$$R + mg = \frac{mv^2}{r}$$

$$R = \frac{mv^2}{r} - mg$$

Minimum speed at top
to continue motion

Put $R = 0$

$$0 = \frac{mv^2}{r} - mg$$

$$0 = m \underline{a_c} + mg$$

$$ma_c = mg$$

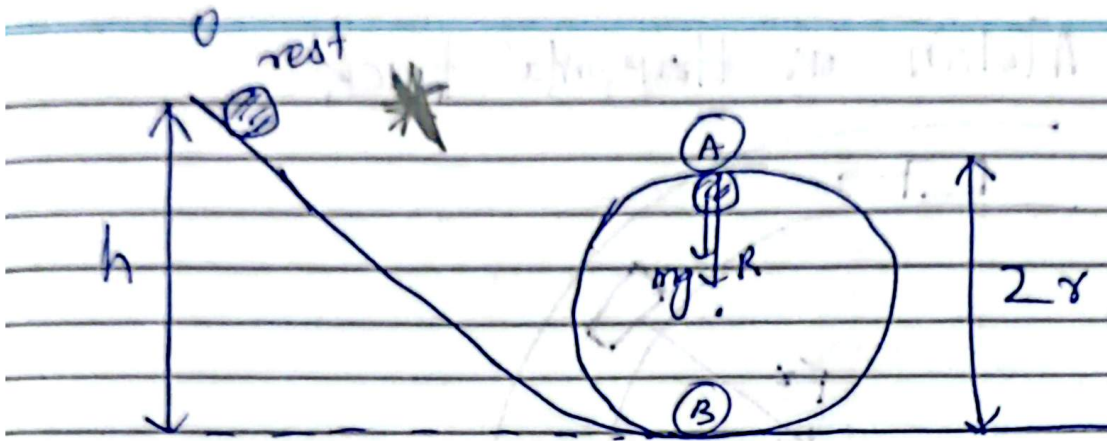
$$a_c > g$$

$$\frac{mv^2}{r} = mg$$

$$v_{\min} = \sqrt{rg}$$

at top

Surface is smooth



~~Max~~ Minimum speed at (A)

$$R + mg = \frac{mv^2}{r} \quad [R=0]$$

$$mg = \frac{mv^2}{r}$$

$$v = \sqrt{rg}$$

$$v_{\min} = \sqrt{rg}$$

min speed at bottom

use energy method

$$\frac{1}{2} m v_B(\min)^2 = mg(2r) + \frac{1}{2} m (\sqrt{rg})^2$$

$$v_B(\min) = \sqrt{5gr}$$

Min height (h)

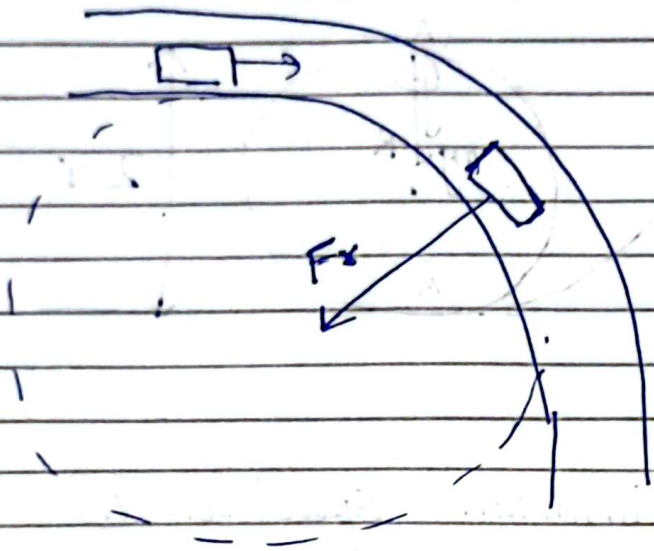
$$T.E(\text{at } O) = T.E \text{ at } B$$

$$mgh = \frac{1}{2} m v_B^2$$

$$gh = \frac{1}{2} (5gr)$$

$$\boxed{h = 2.5r}$$

Motion on Horizontal track



Friction provides centripetal force

$$F_r = \frac{mv^2}{r}$$

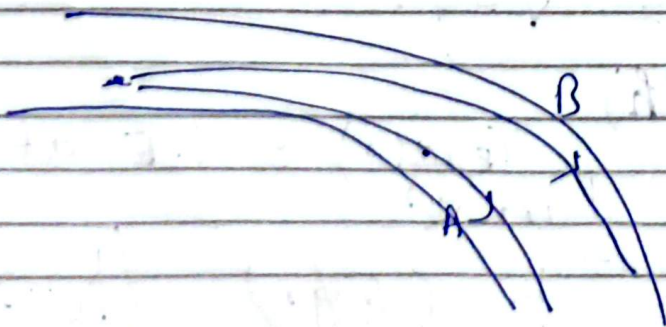
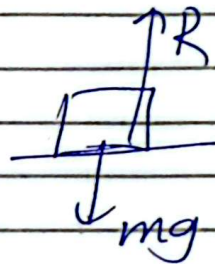
Not in syllabus (UR)

$$\mu R = \frac{mv^2}{r}$$

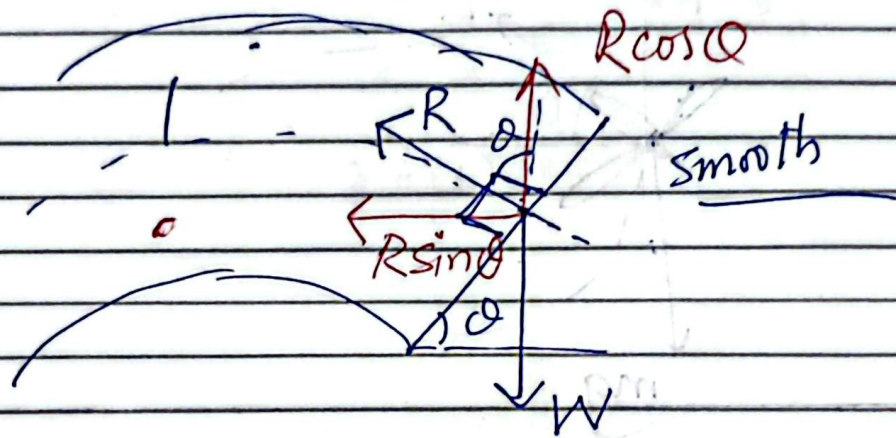
$$\mu mg = \frac{mv^2}{r}$$

$$v^2 = \mu rg$$

$$v_{\max} = \sqrt{\mu rg}$$



Banked Roads



$$R \sin \theta = \frac{mv^2}{r} \rightarrow (1)$$

$$R \cos \theta = mg \rightarrow (2)$$

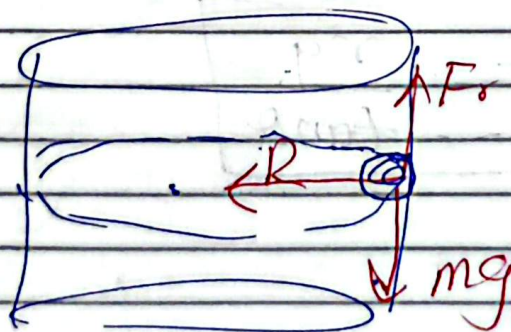
Divide Eq (1) by (2)

$$\tan \theta = \frac{v^2}{rg}$$

$$v_{\max} = \sqrt{rg \tan \theta}$$

$$F_r = \mu R$$

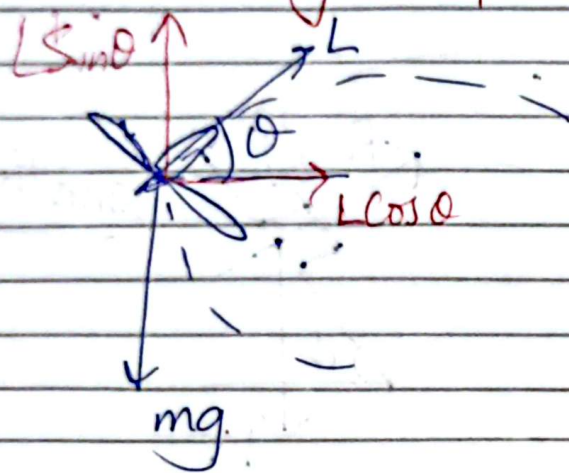
$$R \uparrow \quad F_r \uparrow$$



$$R = \frac{mv^2}{r}$$

$$v \uparrow \quad R \uparrow$$

Turning of aeroplane



$$L \cos \theta = \frac{mv^2}{r} \rightarrow (2)$$

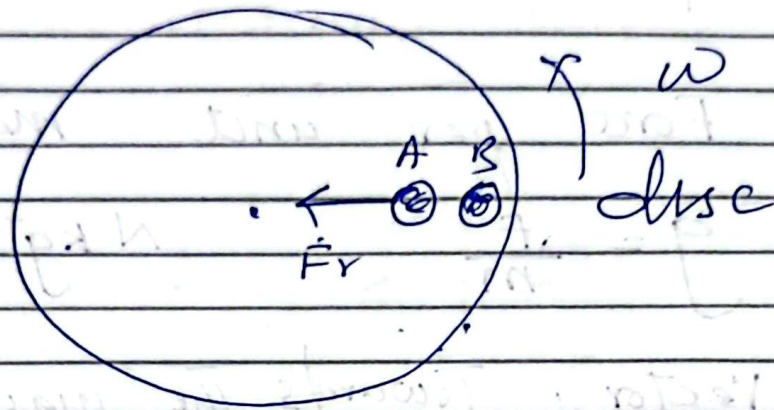
$$L \sin \theta = mg \rightarrow (1)$$

Divide (2) by (1)

$$\frac{L \sin \theta}{L \cos \theta} = \frac{mg}{\frac{mv^2}{r}}$$

$$\tan \theta = \frac{rg}{v^2}$$

$$v^2 = \frac{rg}{\tan \theta}$$



$$F_r = \frac{mv^2}{r}$$

$$F_r = m r \omega^2$$

~~Friction force~~

Centripetal force for B is more than centripetal force for A as ω is same but r is difference.

If ω is increased which object will leave the surface first.

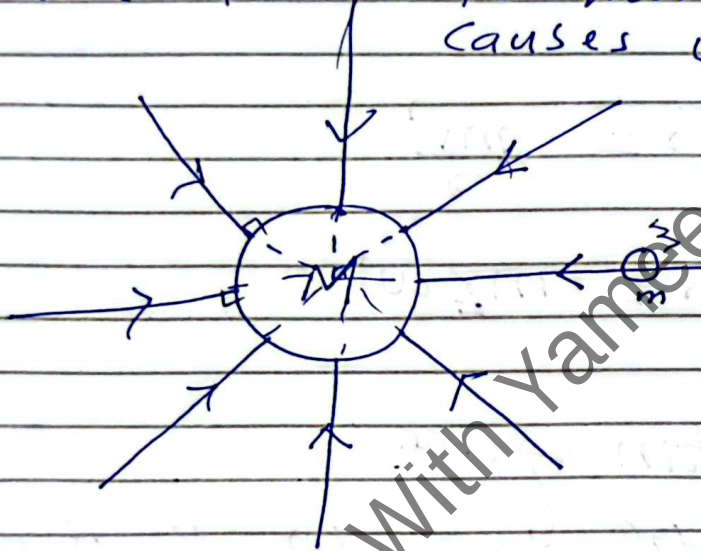
Gravitational Fields

Define gravitational field

Force per unit mass

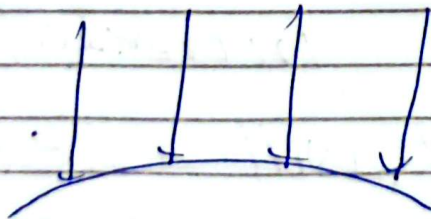
$$g = \frac{F}{m} \quad \text{N kg}^{-1}$$

Vector: Towards the mass that causes it.



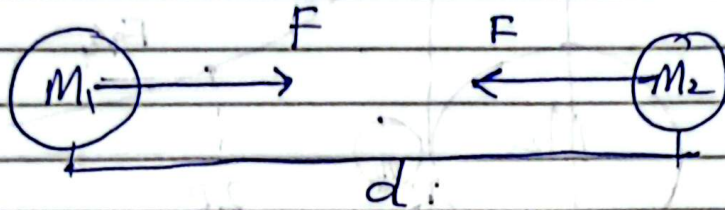
Field is non-uniform as field lines are not parallel.

Near to the surface of earth



field lines are parallel
which show field is uniform
near to the surface of
earth.

Newton's Law of gravitation



Define

"Force between two point masses is directly proportional to product of masses and inversely proportional to square of distance between their centers."

$$F \propto M_1 M_2$$

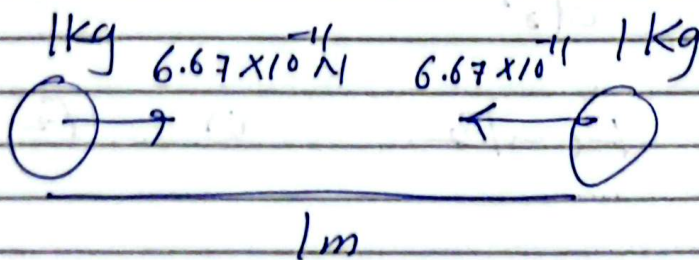
$$F \propto \frac{1}{d^2}$$

$$F \propto \frac{M_1 M_2}{d^2}$$

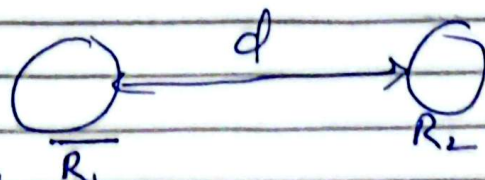
$$F = G \frac{M_1 M_2}{d^2}$$

G : universal gravitational constant

$$G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$$

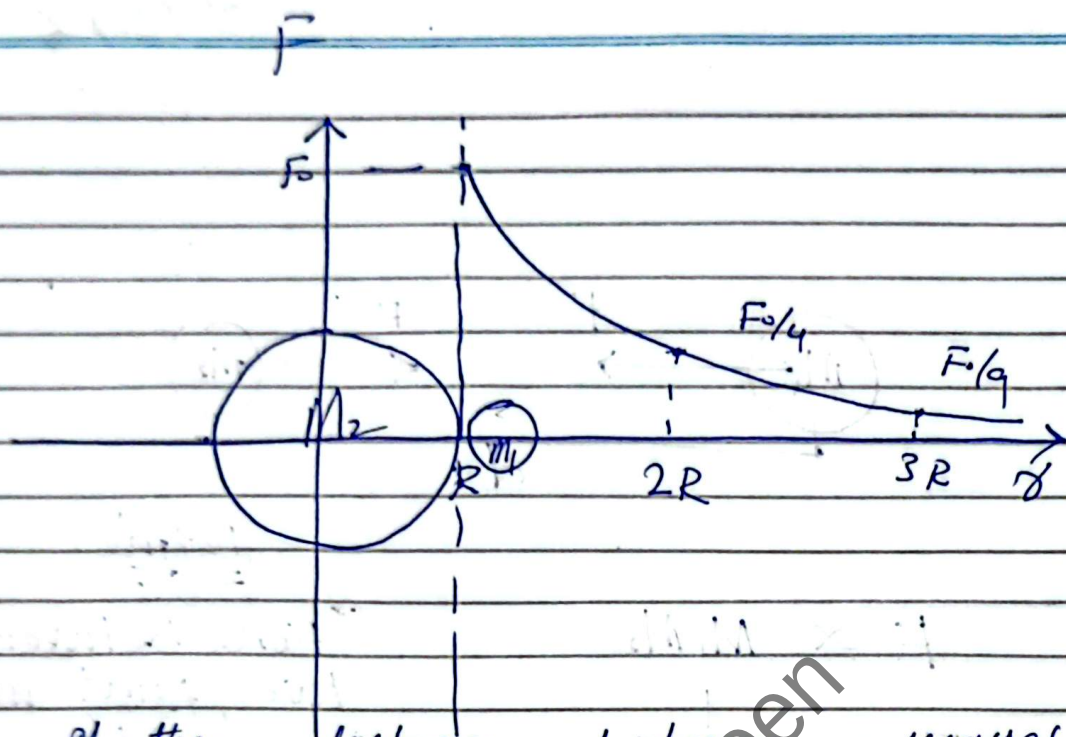


point mass:

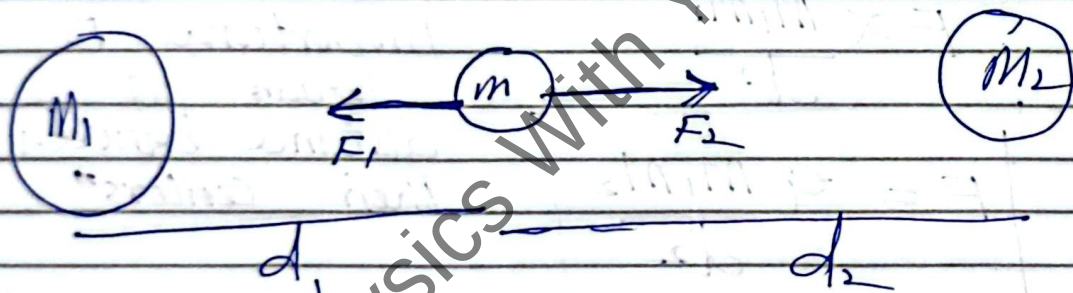


$$\text{if } d \gg R_1 \text{ or } d \gg R_2$$

Summary



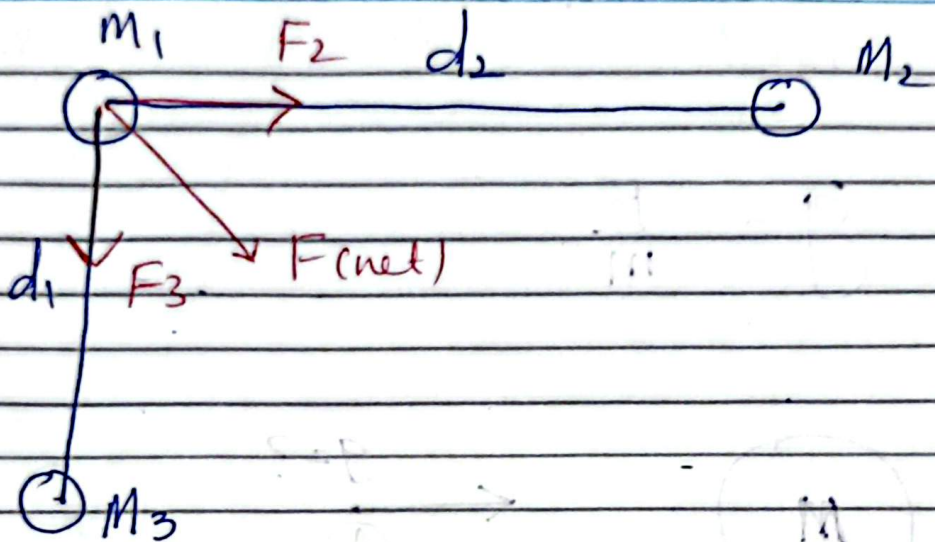
If the distance between masses is increased then force will decrease.



$$F_1 = \frac{q M_1 m}{d_1^2}$$

$$F_2 = \frac{q M_2 m}{d_2^2}$$

$$F_{\text{net}} = F_1 - F_2 \quad \text{OR} \quad F_2 - F_1$$

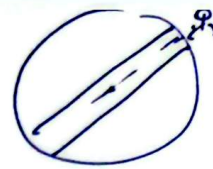
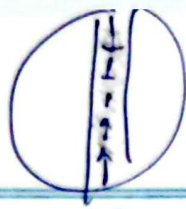


find total force on m_1

$$F_2 = \frac{q m_1 m_2}{d_2^2}$$

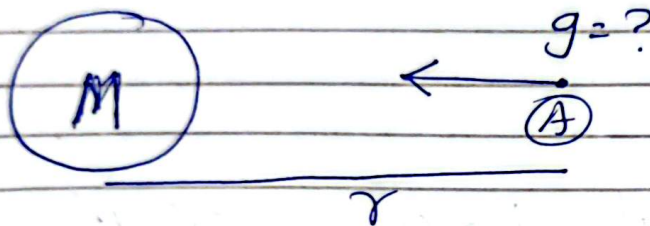
$$F_3 = \frac{q m_1 m_3}{d_1^2}$$

use vector addition rules
to find net force



Gravitational field strength

$$g = \frac{F}{m}$$



Put mass m at A and find force (F)

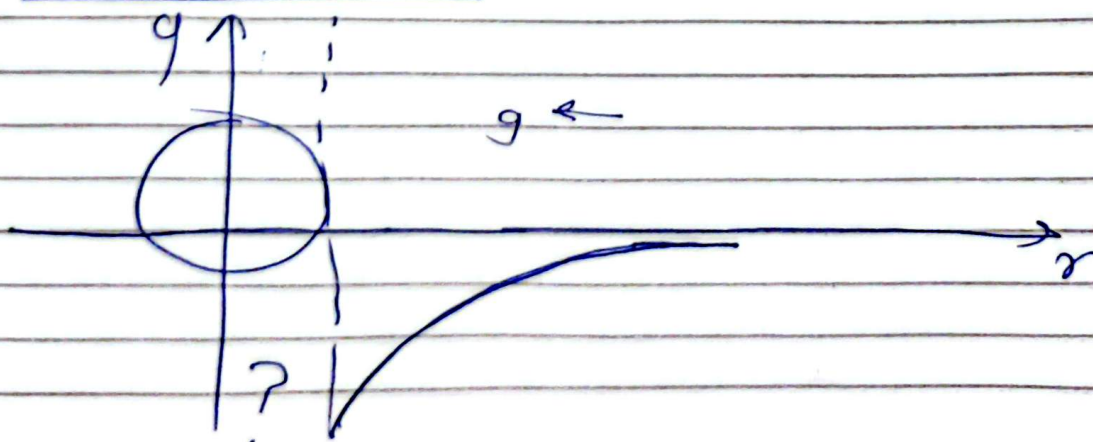
$$F = \frac{GMm}{r^2}$$

Now use $g = \frac{F}{m}$

$$g = \frac{GMm}{r^2} / m$$

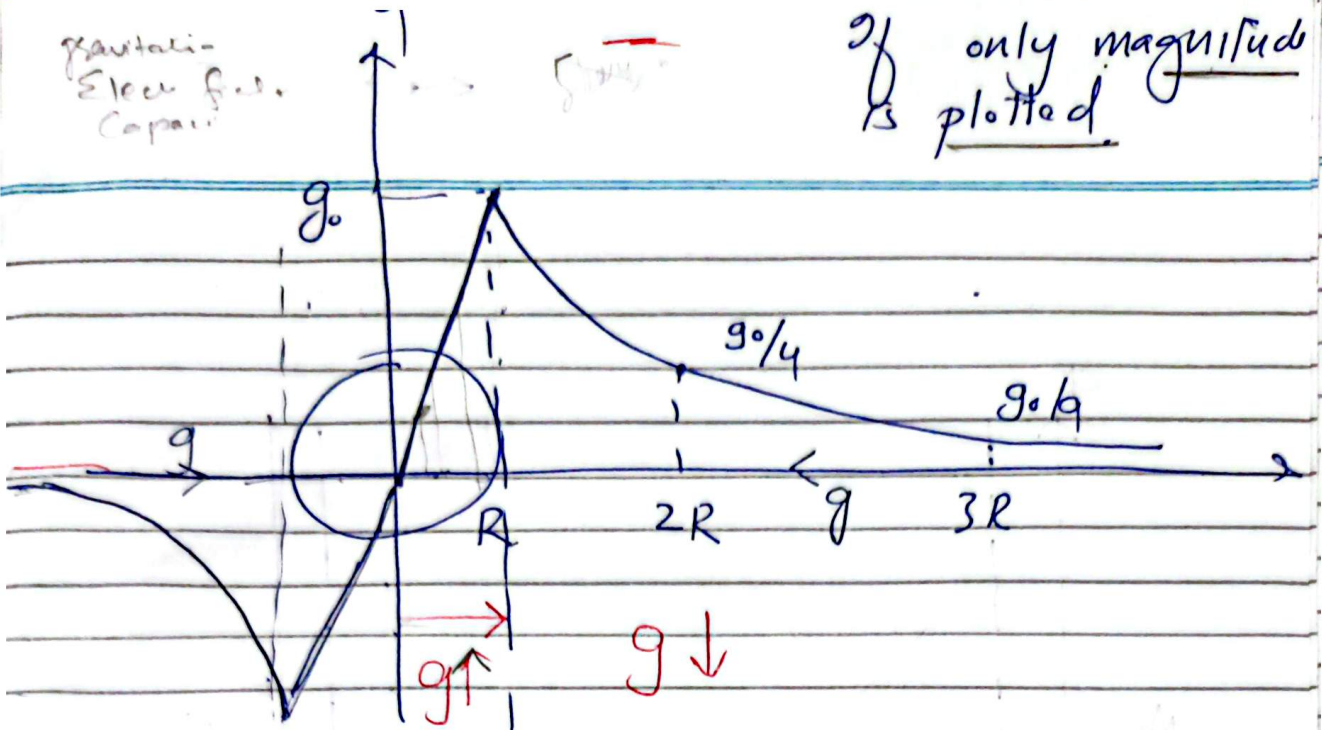
$$g = \frac{GM}{r^2}$$

only applied
away from
 $r \geq R$ surface

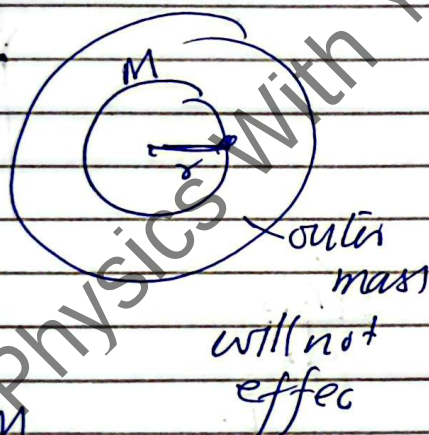


Gravitational
Electrostatic
Capacitance

only magnitude
is plotted



g inside the planet



$$g = \frac{GM}{r^2}$$

$$V = \frac{4}{3} \pi r^3$$

$$g = \frac{G \frac{4}{3} \pi r^3 \rho}{r^2}$$

$$\rho = \frac{M}{V}$$

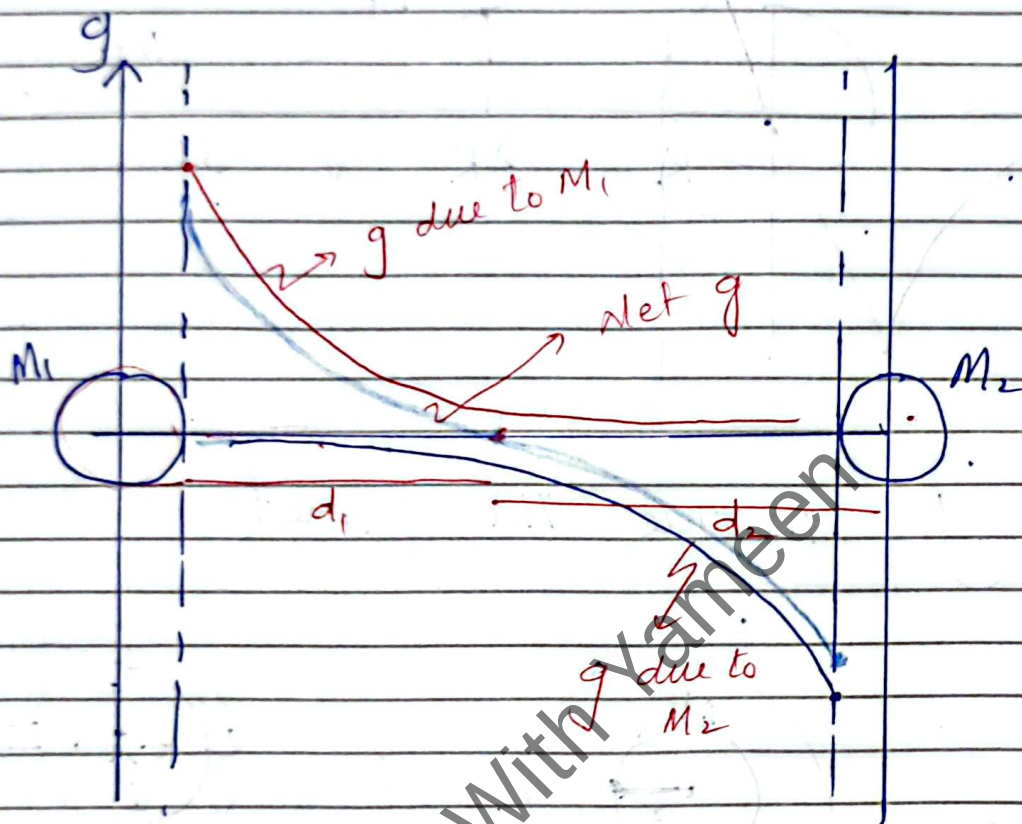
$$g = \left[\frac{4}{3} \pi G \rho \right] r$$

$$M = \rho V$$

$$M = \rho \frac{4}{3} \pi r^3$$

$$g \propto r$$

Gravitational field strength between two masses.



$$\frac{GM_1}{d_1^2} = \frac{GM_2}{d_2^2}$$

$$\frac{M_1}{M_2} = \frac{d_1^2}{d_2^2}$$

$$\frac{d_1}{d_2} = \sqrt{\frac{M_1}{M_2}}$$

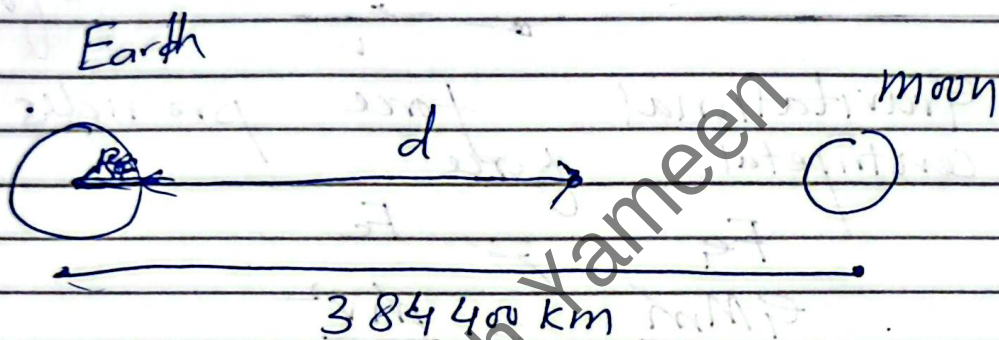
if $M_1 = M_2$ $d_1 = d_2$

if $M_1 > M_2$ $d_1 > d_2$

the point where g is zero

is closer small mass.

Find distance from surface of earth where g is zero due to moon and earth.



$$\frac{GM_E}{(d+R_E)^2} = \frac{GM_M}{(384400-d)^2}$$

$$\frac{GM_E}{d^2} = \frac{GM_M}{(384400-d)^2}$$

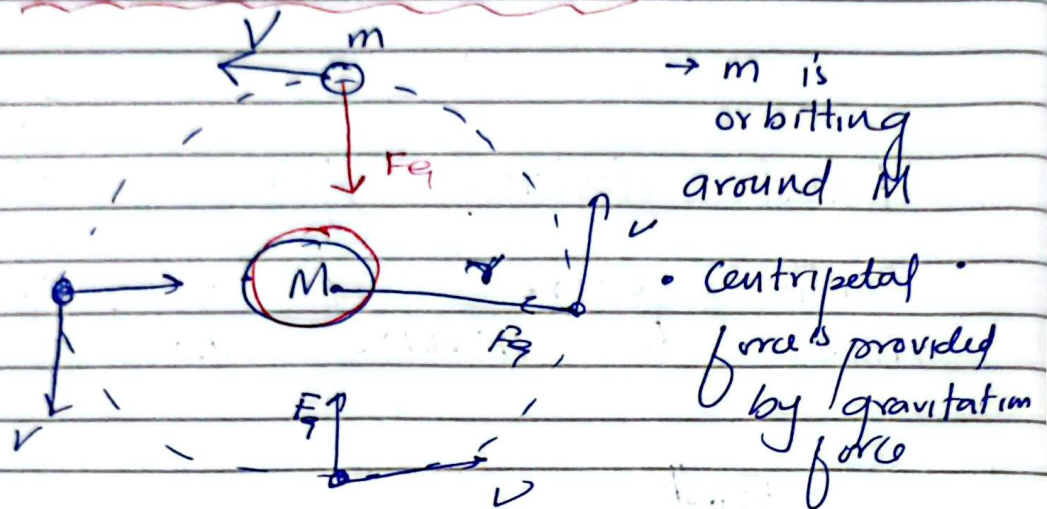
$$\frac{(384400-d)^2}{d^2} = \frac{M_M}{M_E}$$

$$\frac{384400-d}{d} = \sqrt{\frac{M_M}{M_E}}$$

$$\frac{384400-d}{d} = \sqrt{\frac{7.34 \times 10^{22}}{6 \times 10^{24}}}$$

$$\frac{384400-d}{d} = 0.11 \quad d = 346306 \text{ km}$$

Circular orbits



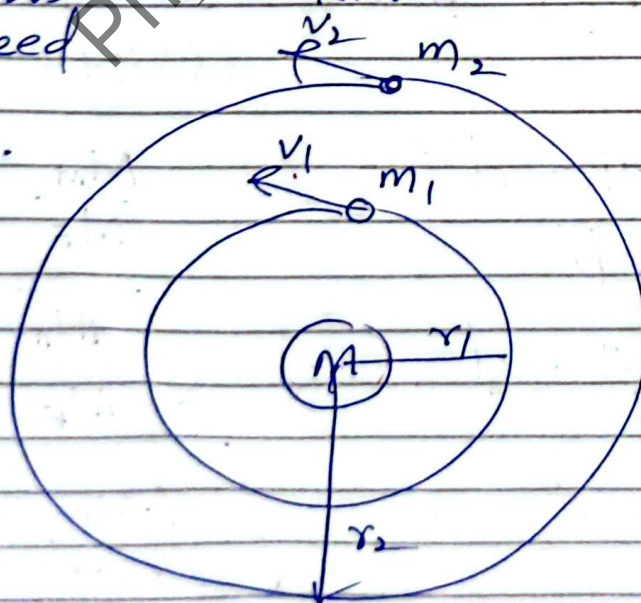
Gravitational force provides centripetal force

$$F_g = F_c$$

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{GM}{r}}$$

This speed is known as orbital speed



$$\frac{v_1}{v_2} = \sqrt{\frac{r_2}{r_1}}$$

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

Relation of ω

$$\frac{GMm}{r^2} = m r \omega^2$$

$$\cancel{\left[\omega^2 \right]} \cancel{\left[\frac{1}{GM} \right]} \left[r^3 \right] GM = r^3 \omega^2 \rightarrow (1)$$

$$r^3 \omega^2 = \text{constant}$$

Kepler's Third Law

$$\omega^2 = \frac{GM}{r^3}$$

for time period

$$\omega = \frac{2\pi}{T} \quad \text{put (1)}$$

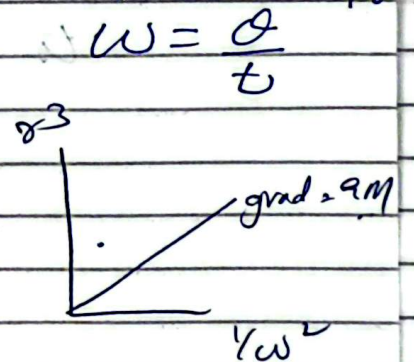
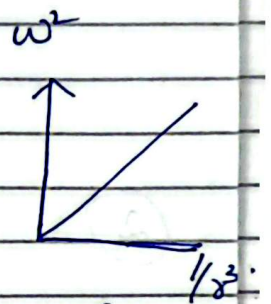
$$GM = r^3 \left(\frac{2\pi}{T} \right)^2$$

$$GM = \frac{r^3 4\pi^2}{T^2}$$

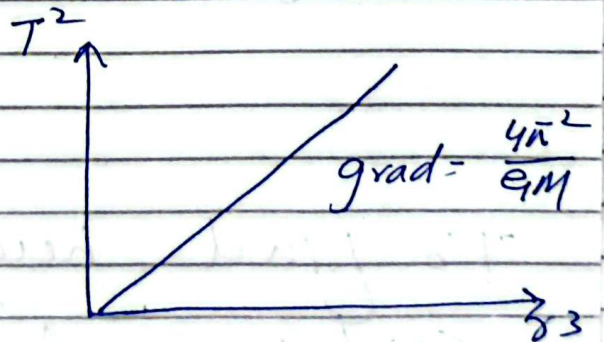
$$\boxed{T^2} = \frac{4\pi^2}{GM} \boxed{r^3}$$

$\downarrow y$ $\downarrow x$

$$T^2 \propto r^3$$



$$\frac{T^2}{r^3} = \text{constant}$$



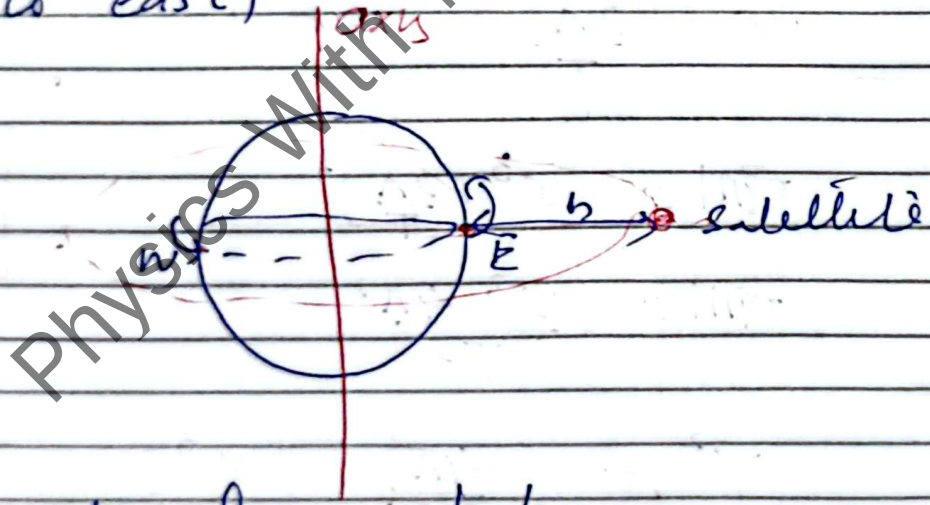
Geostationary Earth Satellite

Geostationary satellite stays at the same position relative to the earth

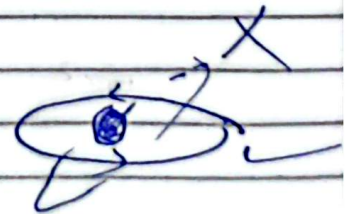
It has following properties

① Time period = 24 hrs
(same as Earth about its axis)

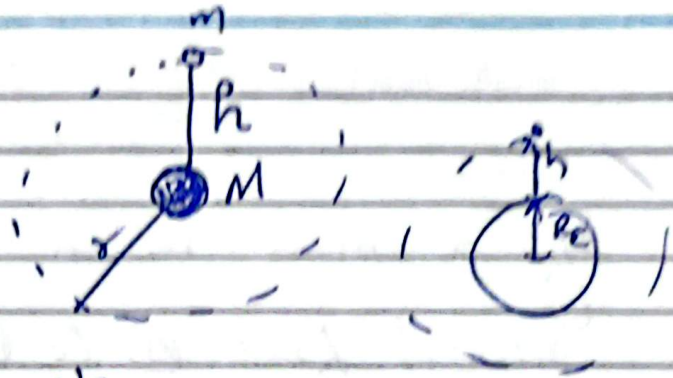
② Direction of rotation is same as the earth (from West to east)



③ Equatorial orbit



To find height from the surface of earth



$$\frac{GMm}{r^2} = mr\omega^2$$

r is distance between earth and moon

$$GM = r^3 \left(\frac{2\pi}{T} \right)^2$$

$$r^3 = \frac{GM}{4\pi^2} T^2$$

$$r^3 = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{4\pi^2} \times (24 \times 3600)^2$$

$$r^3 = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{4\pi^2} \times (24 \times 3600)^2$$

$$r^3 = 7.56 \times 10^{22} \text{ m}^3$$

$$R_E = 6378 \text{ km}$$

$$r = 42283792 \text{ m}$$

$$h + R_E = 4229639$$

$$h = 4229639 - R_E$$

$$h = 4229639 - 6378000$$

$$= 4228379 - 6378000$$

$$h = 35905792 \text{ m} = 35905.792 \text{ km}$$

Gravitational potential energy

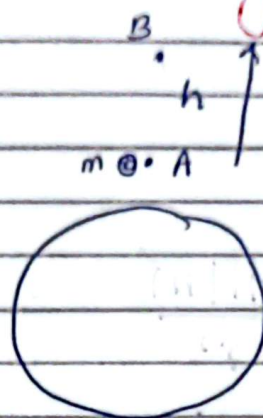
If a mass m is moved from A to B

through height h

$$\Delta E_p = mgh$$

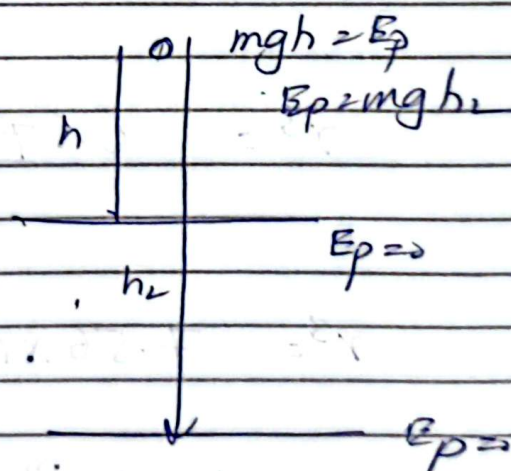
This is only valid when

h is small so that g is not changing b/w A and B.



Now in A2

We will take reference point at infinity (∞)



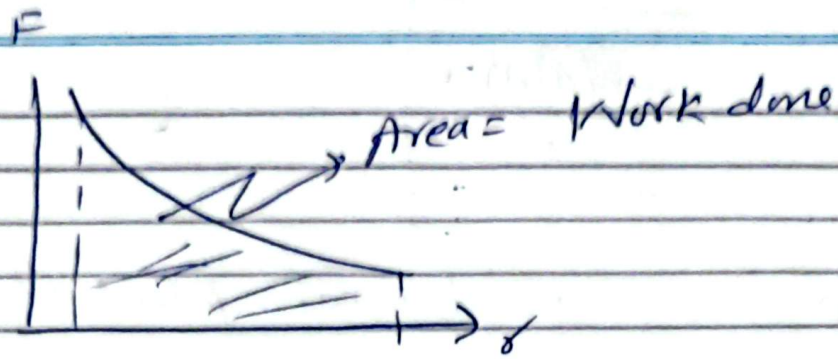
$E_p = ?$



∞

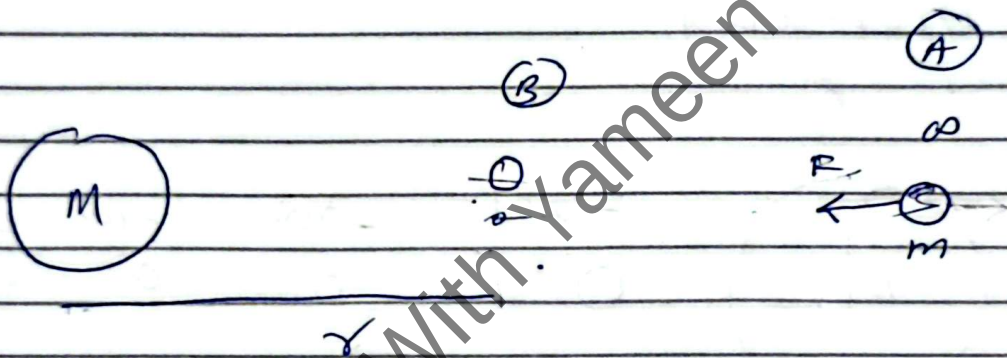


$E_p = 0$



* As Force is not constant
So $W = Fd$ is not applicable

We will take area to ~~the~~
find work done



Work done to move object m from
infinity to point B

$$F = \frac{qMm}{r^2}$$

$$W = \int_{\infty}^r \frac{qMm}{r^2} dr$$

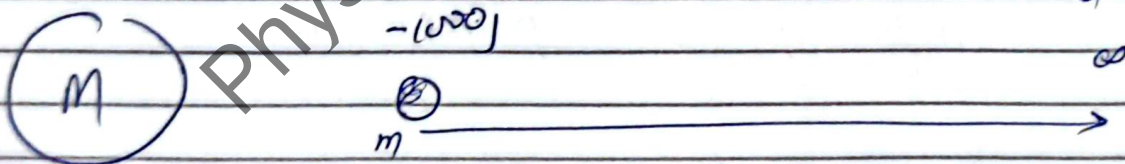
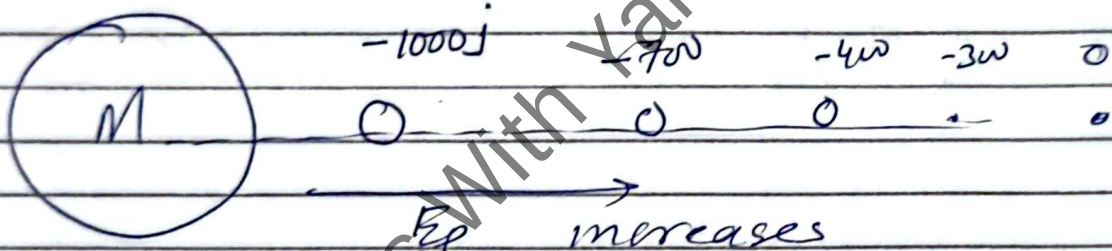
$$E_p = - \frac{qMm}{r}$$

$$E_p = -\frac{GMm}{r}$$

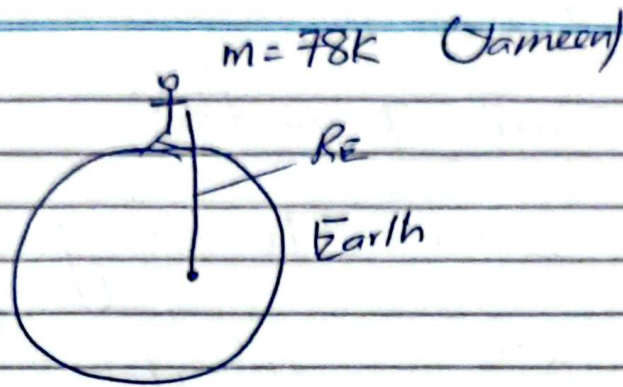
E_p is gravitational potential energy

$E_p = 0$ at ∞

E_p will increase as r is increased



$-1000J$ p.e means
 $1000J$ of energy is
 required to move
 m from A to ∞



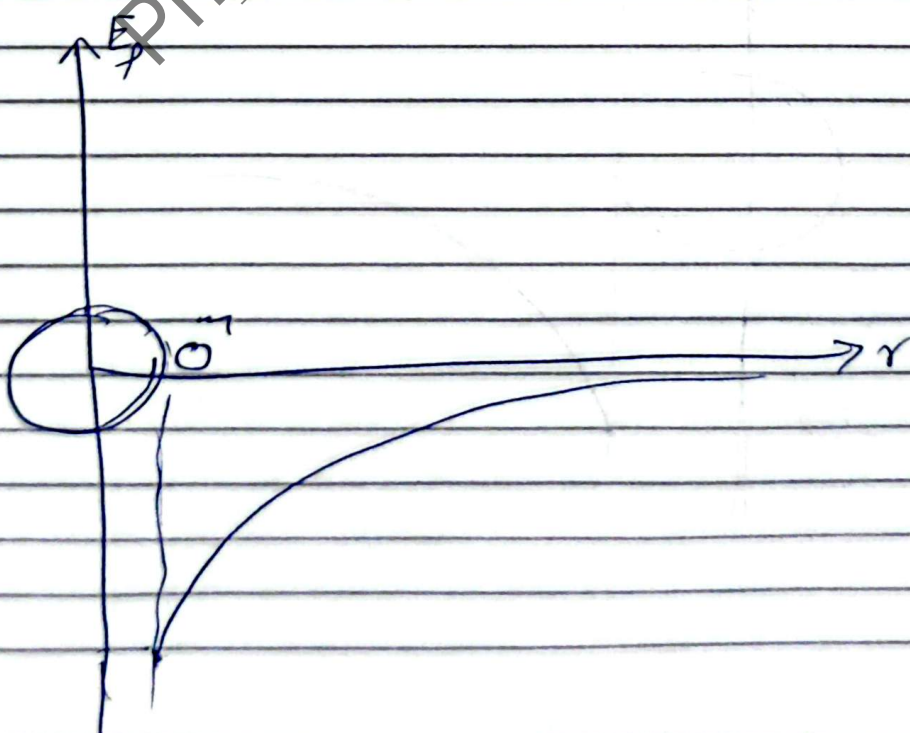
find Gravitational potential energy of
sir Jameen at the surface of earth.

$$E_p = - \frac{GMm}{r}$$

$$R_E = 6400 \text{ km}$$

$$= \frac{-6.67 \times 10^{-11} \times 6 \times 10^{24} \times 78}{6400 \times 10^3 \text{ m}}$$

$$E_p = -4 \times 10^3 \text{ J}$$



ϕ

Gravitational potential

Work done to bring a unit test mass from infinity to a point inside gravitational field is known as gravitational potential.

$$E_p = -\frac{GMm}{r}$$

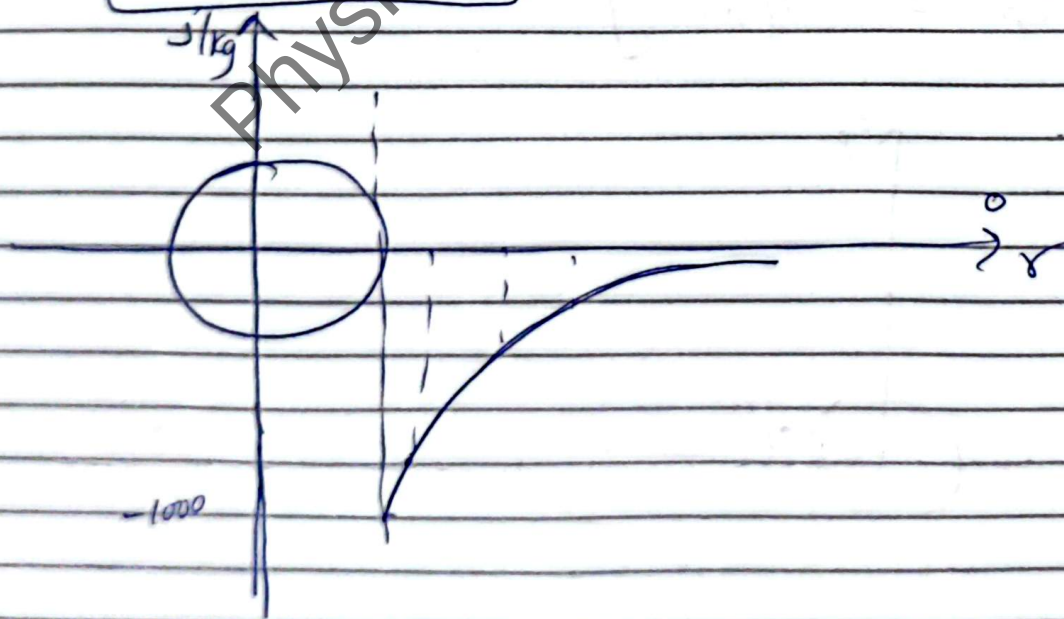
$$\phi = \frac{E_p}{m}$$

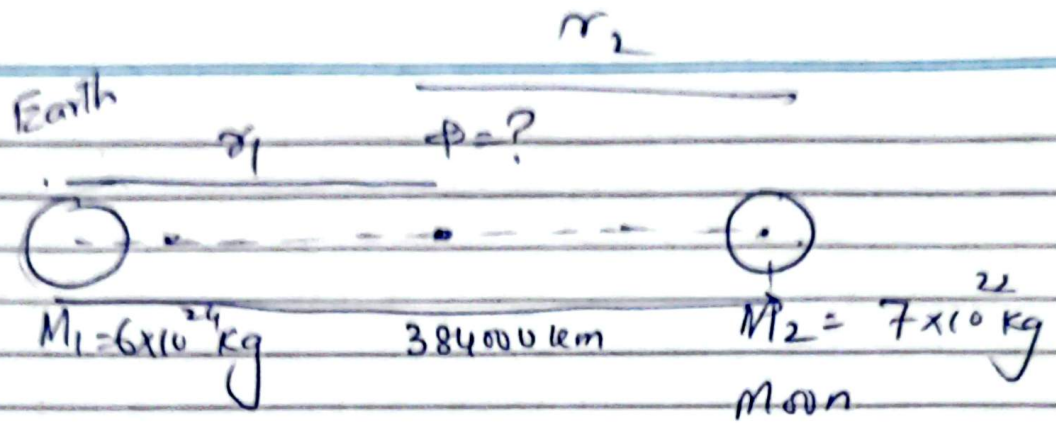
$$\phi = -\frac{GM}{r}$$

$$\phi = \frac{W}{m}$$

$$\phi = \frac{\text{J}}{\text{kg}}$$

unit
scalar





$$R_E = 6400 \text{ km}$$

total

b/w

find potential at mid point of
Earth and moon.

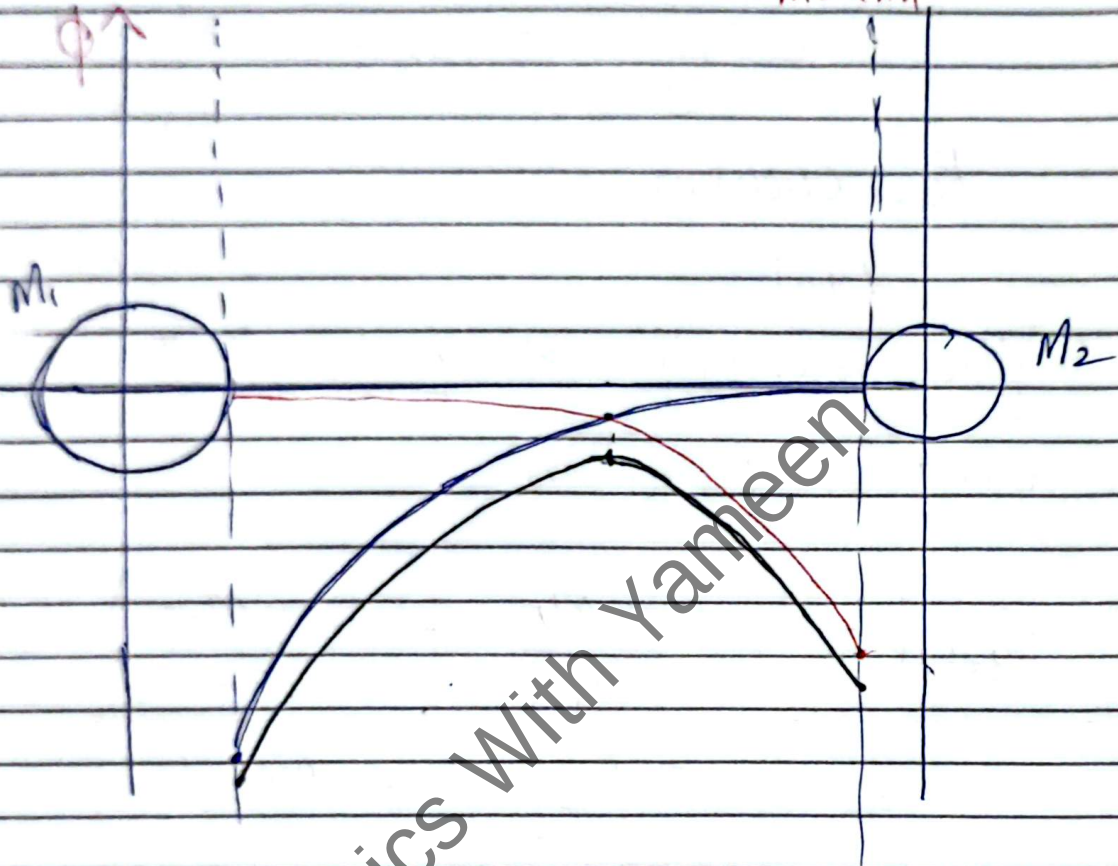
$$\begin{aligned} \phi_{\text{total}} &= -\frac{GM_E}{r_1} + \left(-\frac{GM_M}{r_2}\right) \\ &= -G \left[\frac{M_E}{r_1} + \frac{M_M}{r_2} \right] \\ &= -6.67 \times 10^{-11} \left[\frac{6 \times 10^{24}}{192000 \times 10^3} + \frac{7 \times 10^{22}}{192000 \times 10^3} \right] \end{aligned}$$

$$\phi = -\frac{GM}{r}$$

$$g = \frac{GM}{r^2}$$

Graphs of Net gravitational potential b/w two masses

$$M_2 < M_1$$



—: Due to M_1

—: Due to M_2

—: Net ϕ

Relation b/w gravitational field strength and gravitational potential

$$g = \text{gradient}(\phi)$$

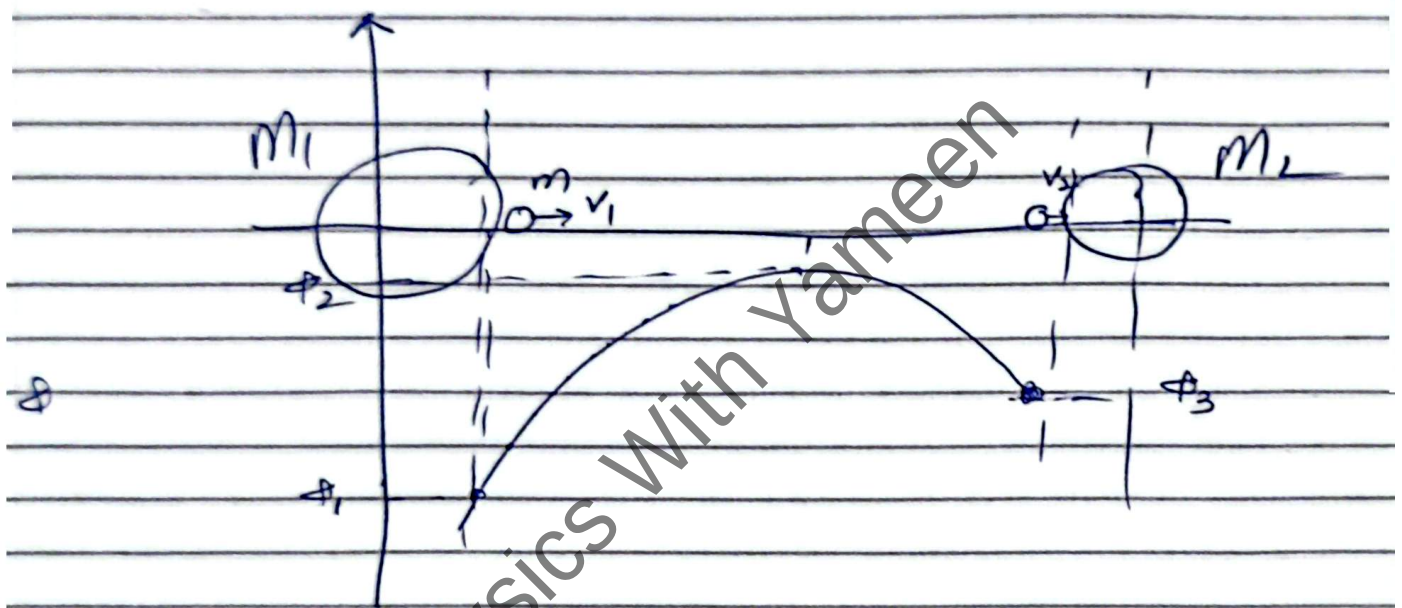
$$-\frac{GMm}{r^2}$$

$$-\frac{GMm}{r^2}$$

Relation b/w potential and potential energy

$$E_p = m \phi$$

$$\Delta E_p = m \Delta \phi$$



m kg moves from surface of

M_1 to the surface of M_2

$$\Delta E_p = m (\phi_3 - \phi_1)$$

$$\Delta E_p + \Delta E_k = 0$$

Law of conservation of energy.

$$\Delta E_k = -\Delta E_p$$

$$\Delta E_k(\max) = m (\phi_2 - \phi_1)$$

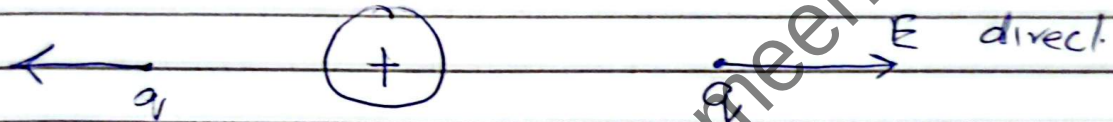
Electric field

Electric field

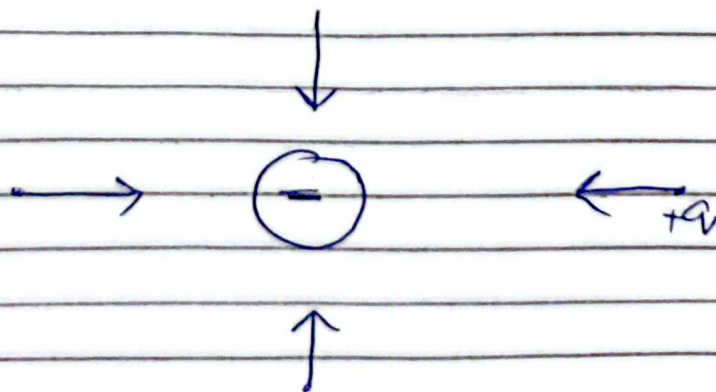
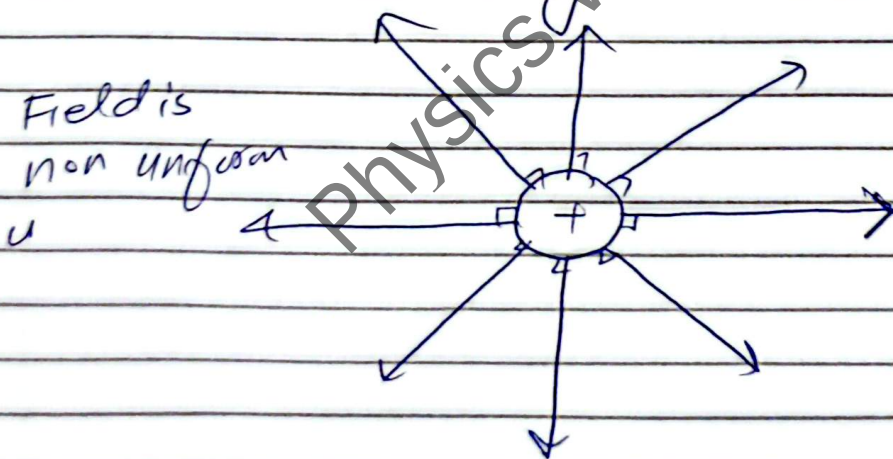
Electric force per unit positive charge

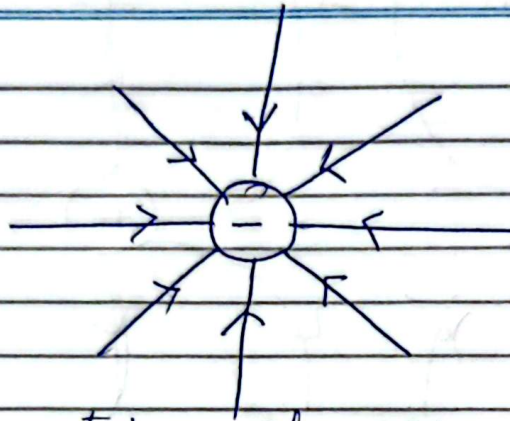
$q = +ve$ $E = \frac{F}{q}$ $N\ C^{-1}$ $F = mg$ $F = Eq$

Vector

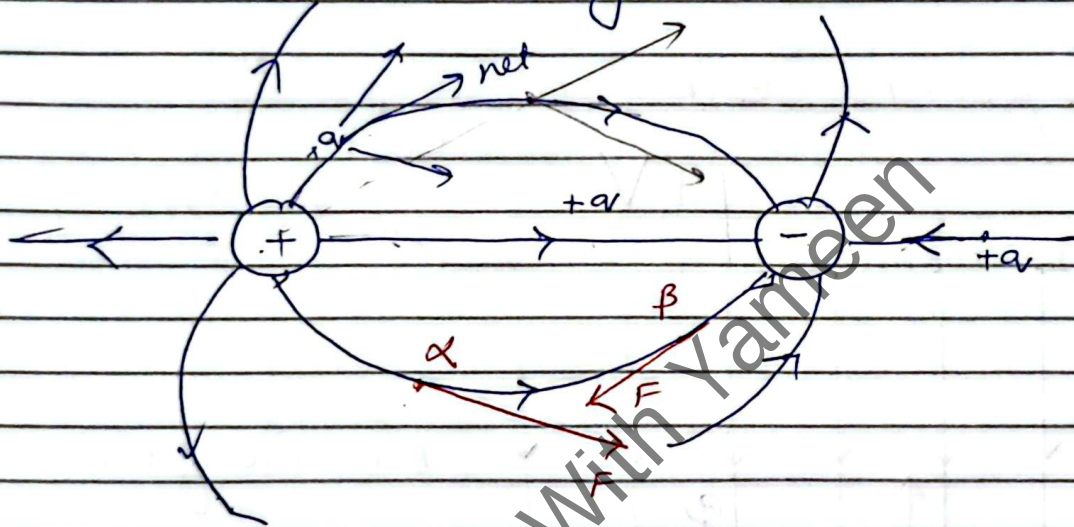


Direction: away from positive charge.





Due to two charges

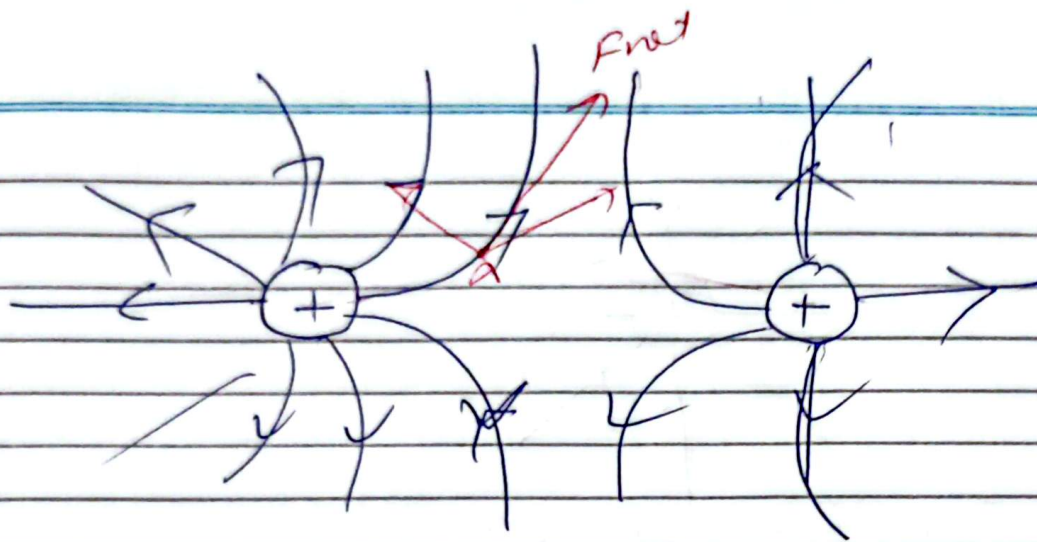


Force on any charge particle
acts at tangent to the
curve.

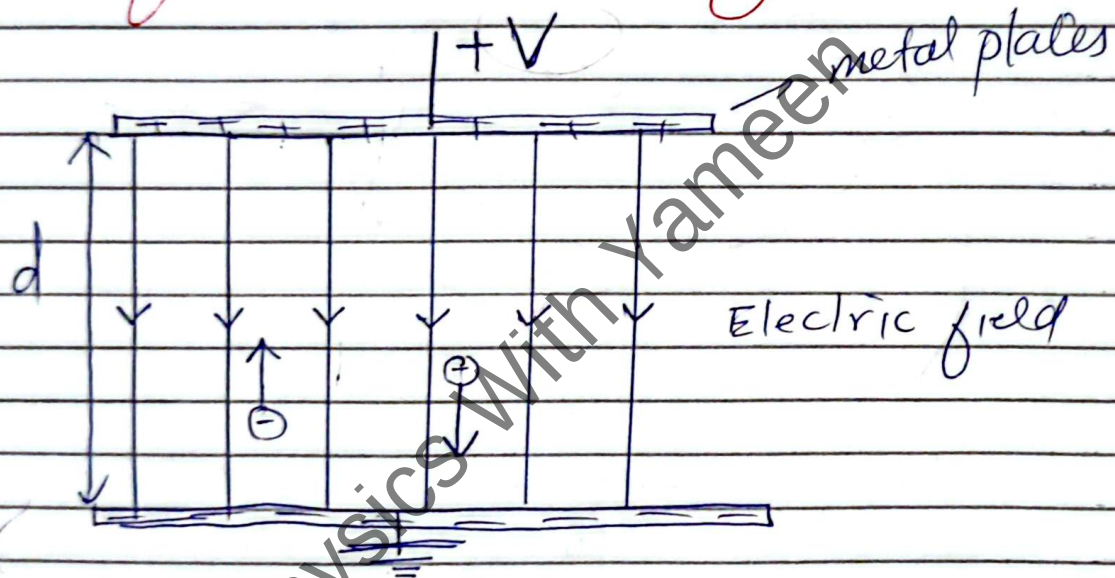
in ~~an~~ electric field direction
of charge is +ve

in opposite direction of charge
is negative

as shown forces on α and β.



uniform electric field



parallel line $\rightarrow$ uniform Electric field

Field strength: Depends on

- i) potential difference
- ii) distance b/w plates

$$E = \frac{V}{d}$$

Field strength

V : potential difference

proof is
not in
the
syllabus

$$E = \frac{F}{q}$$

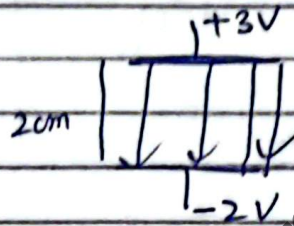
$$V = \frac{W}{q}$$

$$E = \frac{1 \times \frac{V}{W} \cdot W}{W} = \frac{V}{d}$$

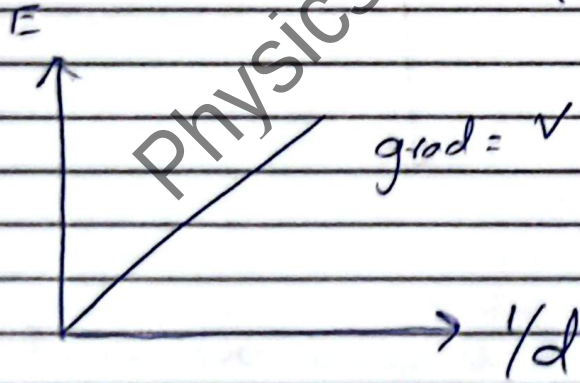
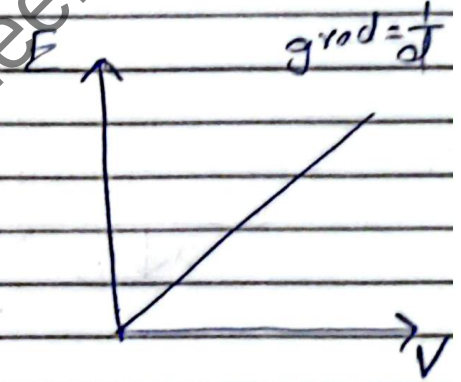
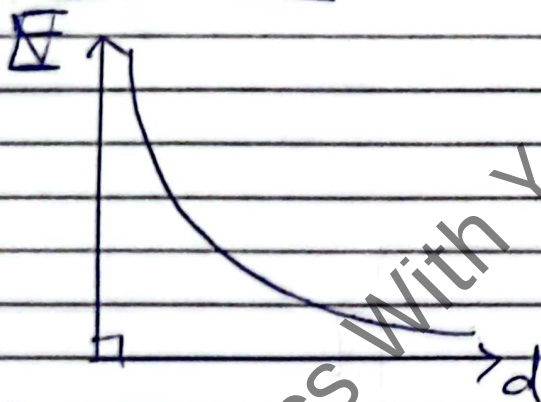
$$V = \frac{Ed}{\epsilon_0}$$

$$E = \frac{V\epsilon_0}{d}$$

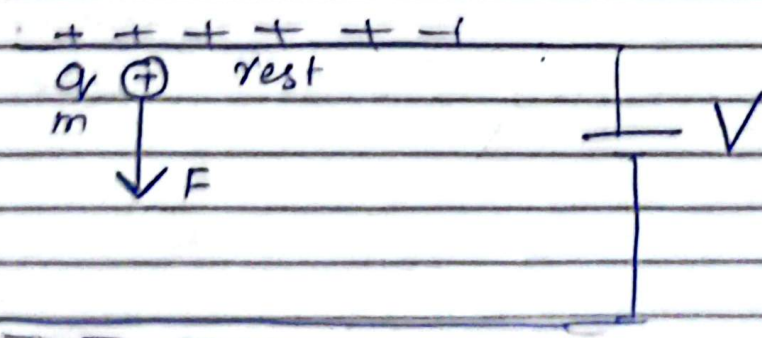
$$E = \frac{V}{d}$$



$$E = \frac{5}{2 \times 10^{-2}} = 250$$



Ignore gravity



$$F = Eq$$

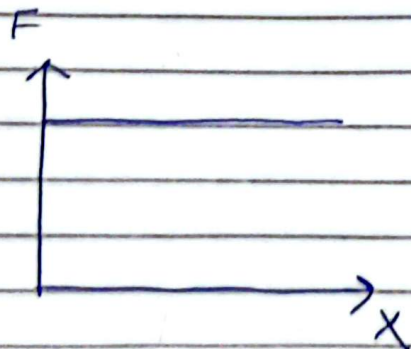
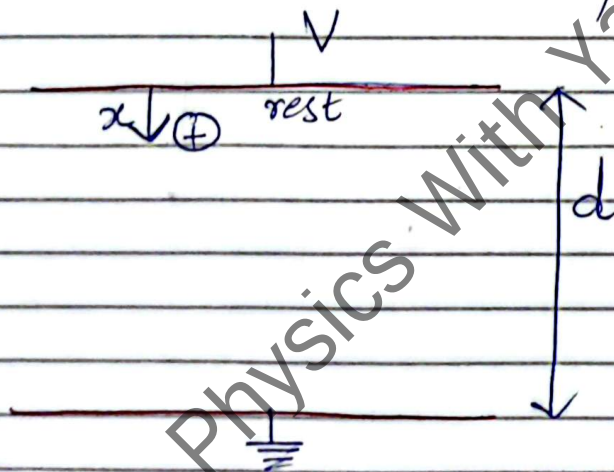
$$F = \frac{V}{d} q$$

$$ma = \frac{Vq}{d}$$

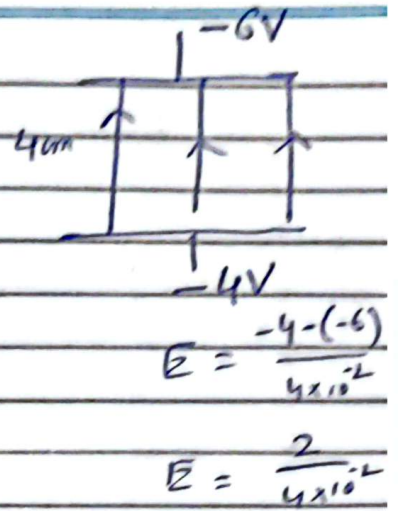
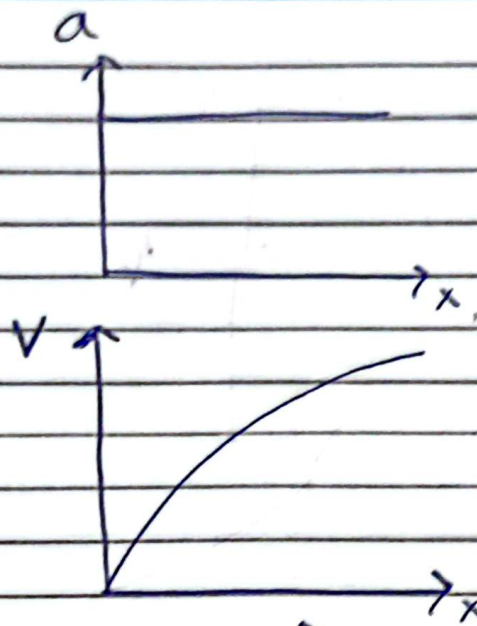
$$a = \frac{Vq}{dm}$$

a is constant

so equations of motion are applicable.



x : distance of charged particle from upper plate



$$v^2 = u^2 + 2ax$$

$$u = 0$$

$$v^2 = 2ax$$

$$2ax = v^2 - u^2$$

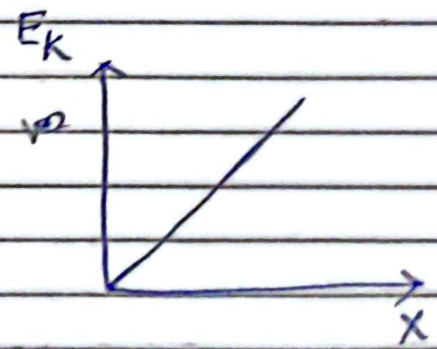
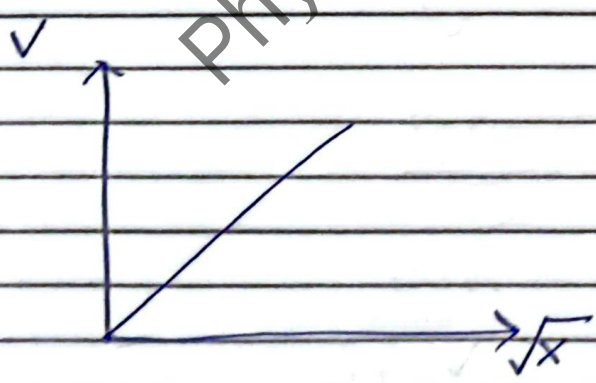
$$v = \sqrt{2ax}$$

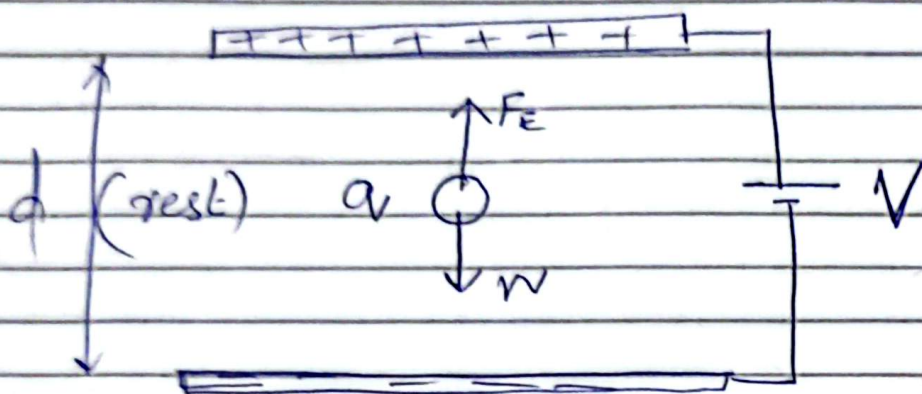
$$v = \sqrt{2ax}$$

$$v = \sqrt{2a} \sqrt{x}$$

$$\int \frac{dv}{dx} = \sqrt{2a} \times \frac{1}{2} x^{-\frac{1}{2}}$$

$$\frac{dv}{dx} = \frac{\sqrt{2a}}{2\sqrt{x}}$$





if charge is negative The
Electric force $\rightarrow$ up

gravitational " $\rightarrow$ Down

if at rest

$$F_E = W$$

$$Eq = mg$$

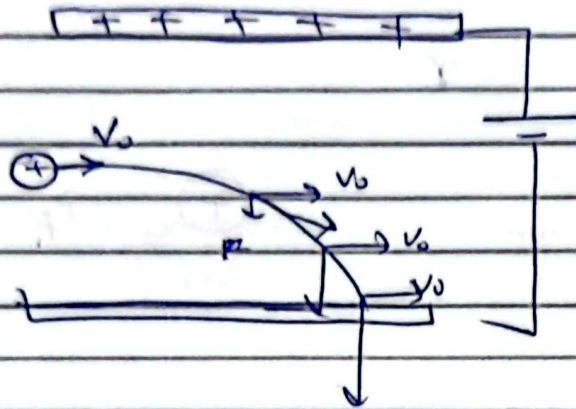
$$q = \frac{mg}{E}$$

$$q = \frac{mg}{V/d}$$

$$q = \frac{mgd}{V}$$

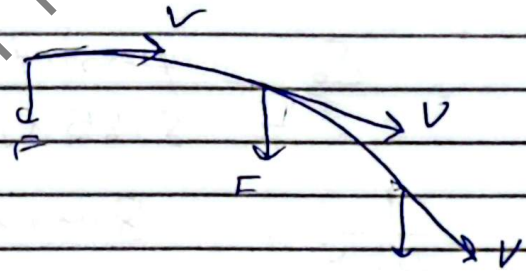
[if V is increased now

Charge q will accelerate up]



it will follow projectile motion

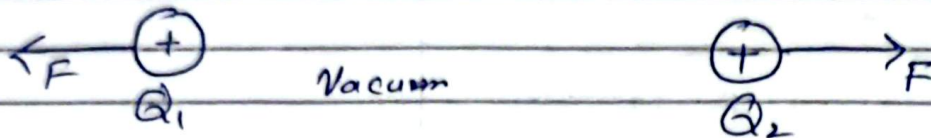
It is not an arc of circle
because Force is not $\perp$ to
Velocity



Coulomb's Law

F: electric force

r



$$F \propto \frac{Q_1 Q_2}{r^2}$$

$$F = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2}$$

$Q_1 \rightarrow \text{coulomb}$

$Q_2 \rightarrow "$

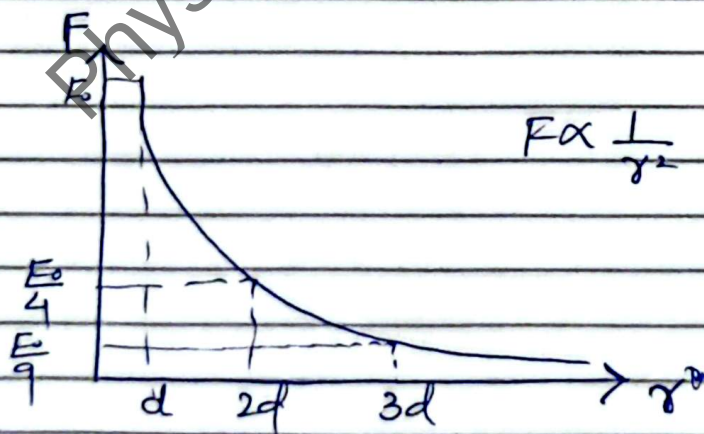
$r \rightarrow \text{m}$

$F \rightarrow \text{N}$

$\epsilon_0 =$ permittivity of free space
 $= 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$

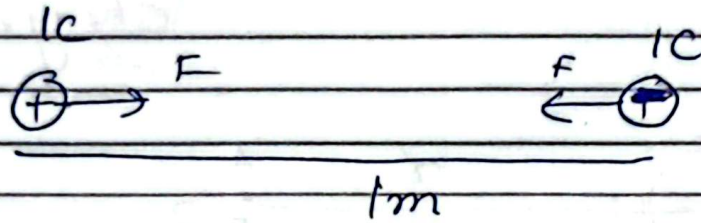
ϵ_0 free space

$$\frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2$$



Define: Force b/w Two point charges is directly proportional to product of their charges and inversely proportional to

square of distance b/w them



$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q_1 Q_2}{r^2}$$

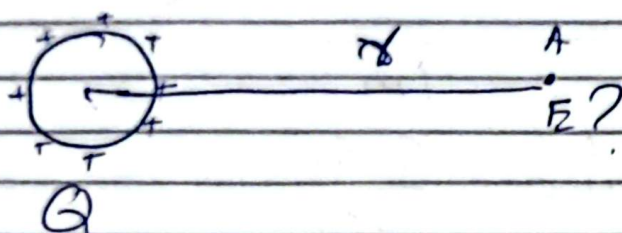
$$F = 8.99 \times 10^9 \times \frac{1 \times 1}{1^2}$$

$$F = 8.99 \times 10^9 \text{ N}$$

$$Q = \frac{F}{E}$$

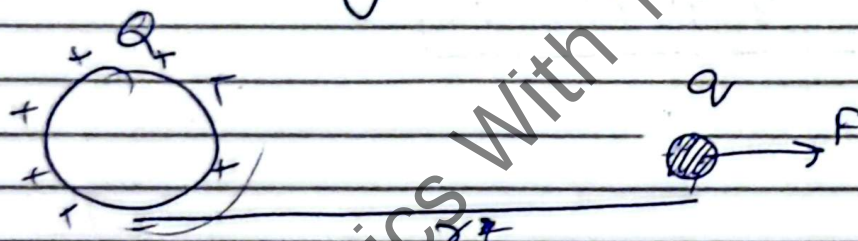
Electric field strength

$$E = \frac{F}{q} \quad \text{①} \quad \text{electric force per unit positive charge}$$



E at point A ?

put charge q at point A



$$F = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r^2}$$

use eq ①

$$E = \frac{F}{q}$$

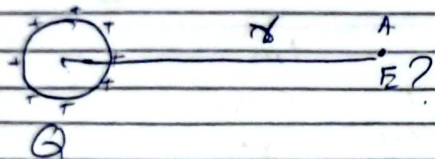
r , distance from centre

$$= \frac{1}{4\pi\epsilon_0} \frac{Qq}{r^2} / q$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

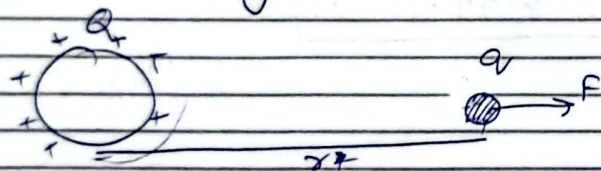
Electric field strength

$$E = \frac{F}{q} \quad \text{① electric force per unit positive charge}$$



E at point A ?

put charge q at point A



$$F = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r^2}$$

use eq ①

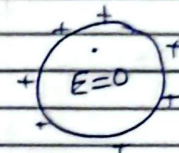
$$E = \frac{F}{q}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

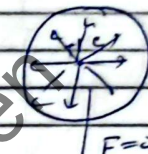
$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

r, distance from centre

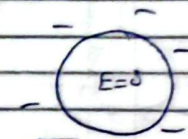
Inside Metal Charged sphere



charge lies only on surface

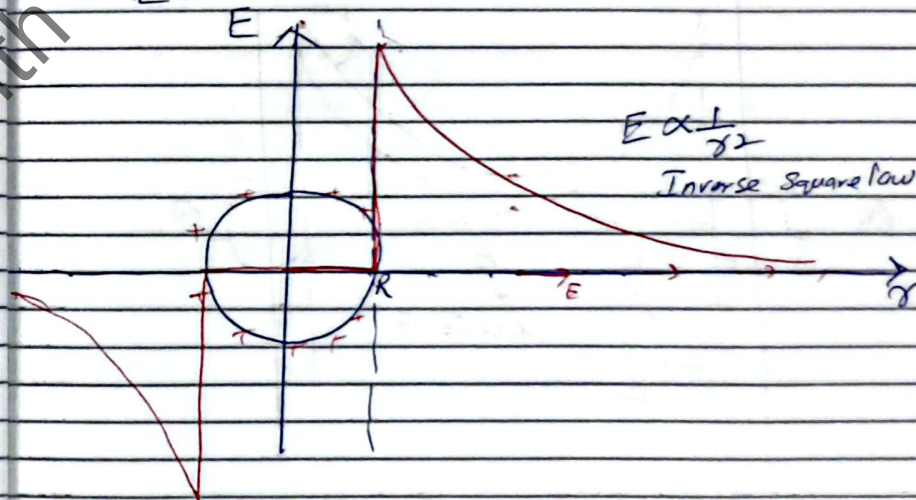


F=0 so E=0



Inside metal charged sphere

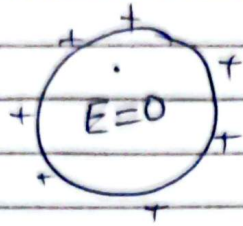
E=0



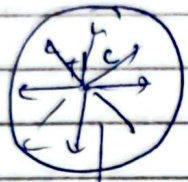
Inside

Metal

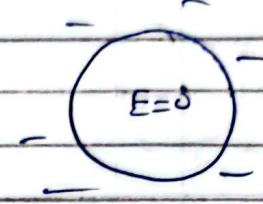
Charged sphere



charge lies only on surface



$F=0$ so $E=0$



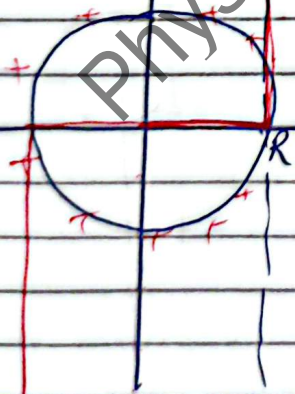
Inside metal charged sphere

$E=0$

E

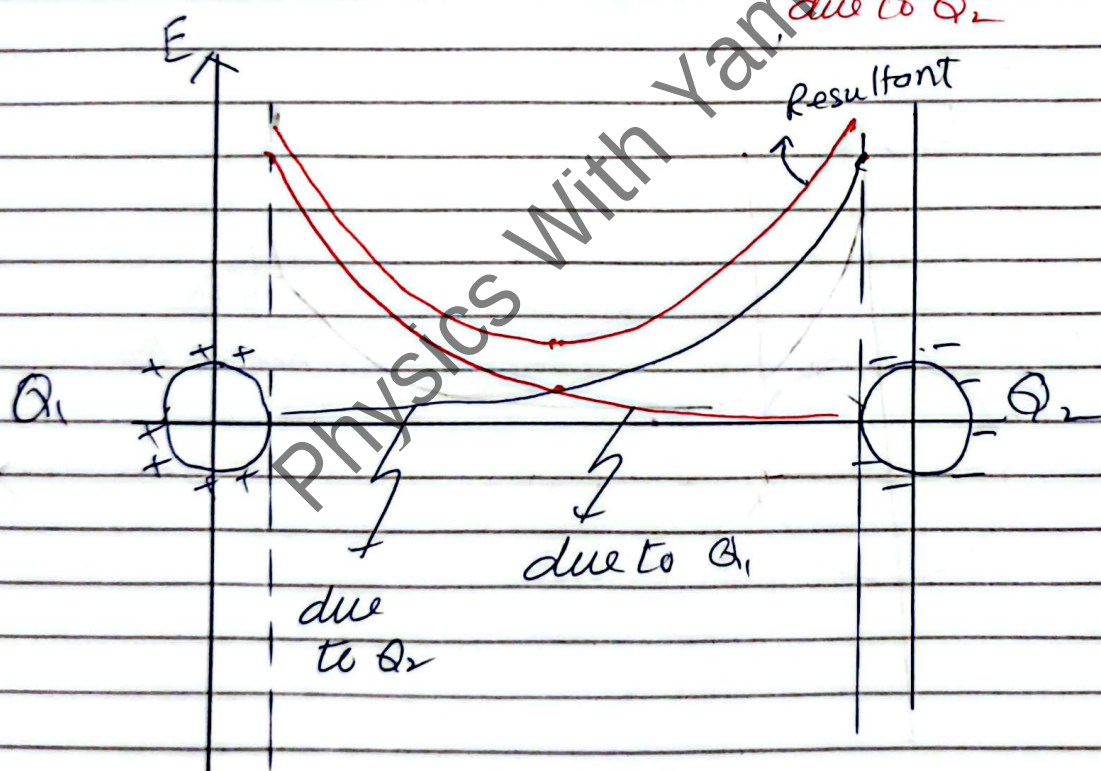
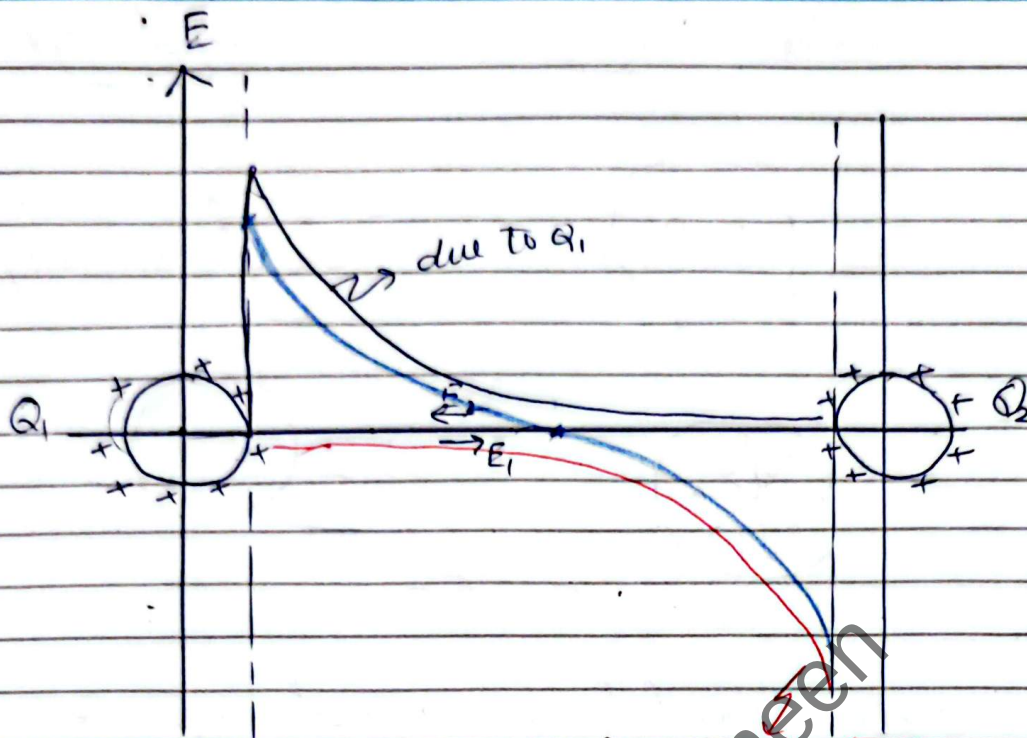
$E \propto \frac{1}{r^2}$

Inverse Square law

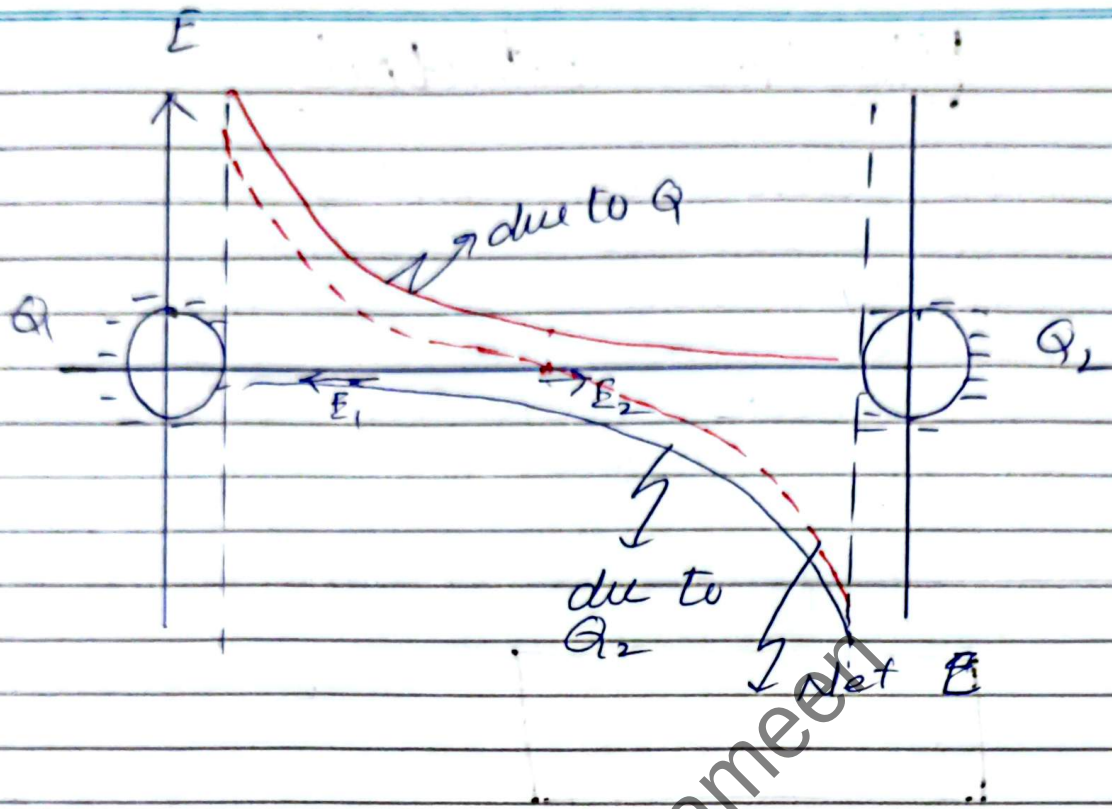


E

r



E at any point b/w charges
 due to Q_1 $\rightarrow$ Right
 " Q_2 $\rightarrow$ Right
 Same direction



Physics With Yameer

Electric potential

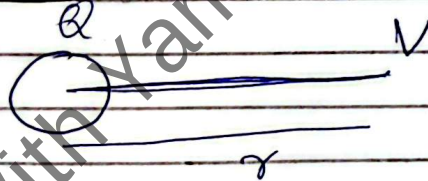
Work done to bring a unit positive charge from infinity to a point inside gravitational/electric field is called electric potential of that point.

Formula

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

$$V = \frac{W}{Q}$$

unit: J/C
scalar



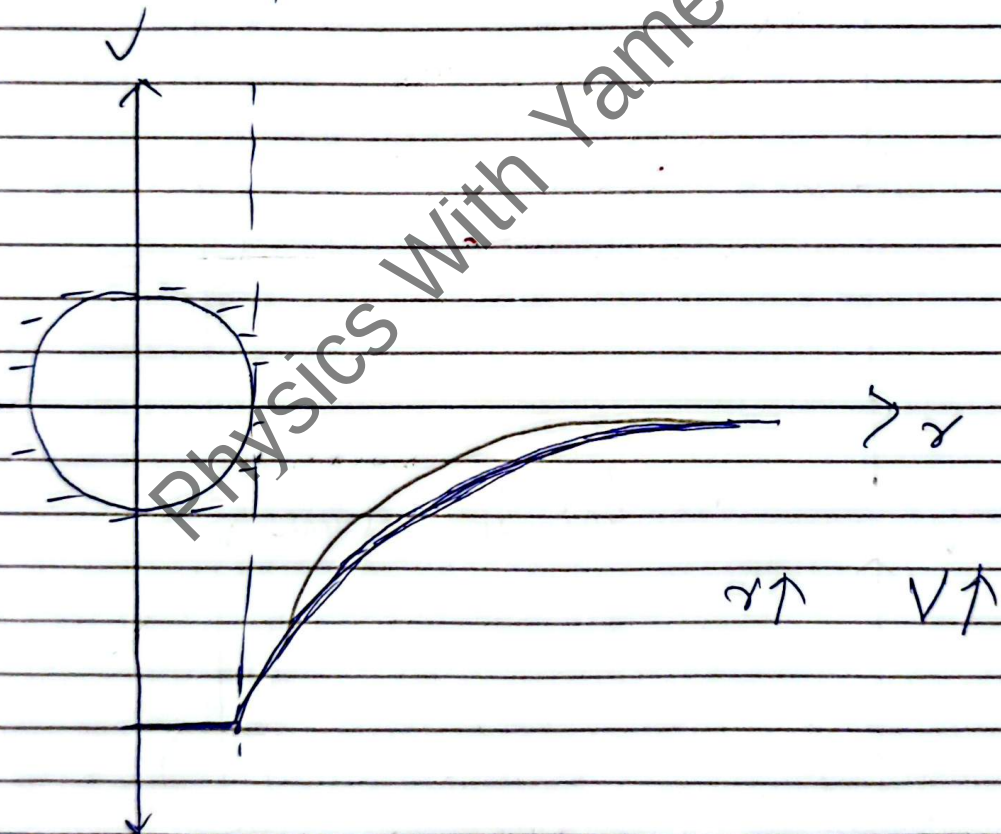
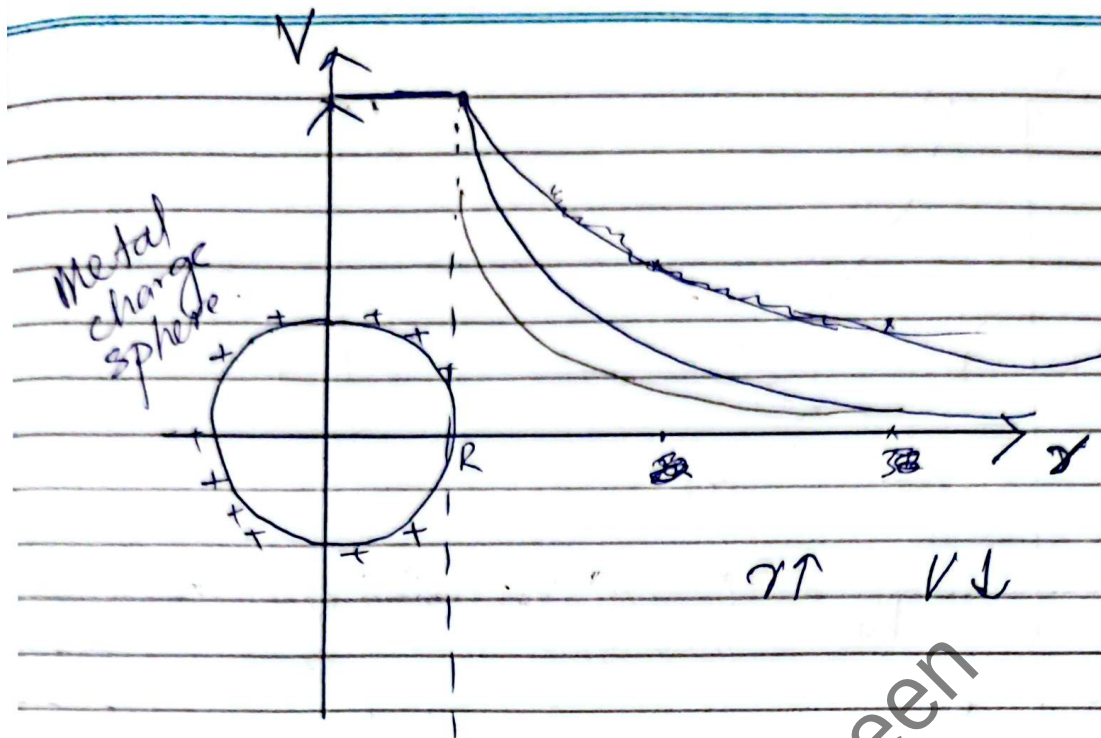
Electric potential can be negative and positive.

* If Q is positive then V is positive

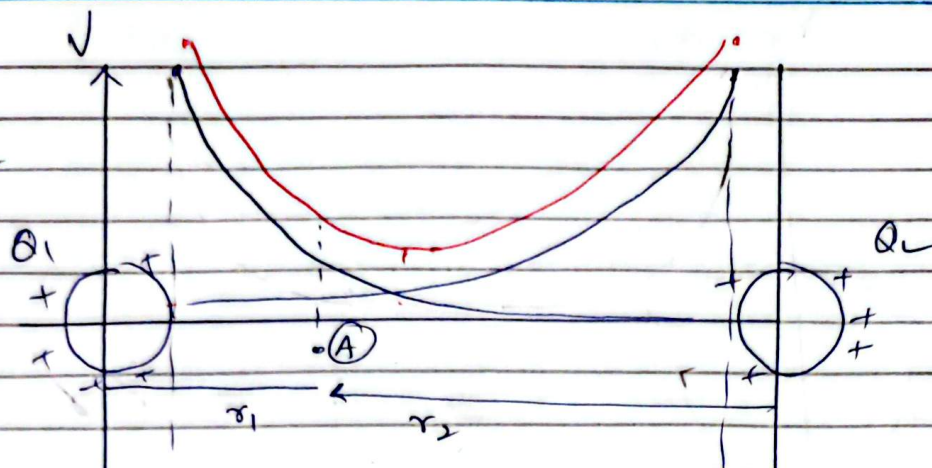
* If Q is negative then V is negative

* V is zero at infinity.

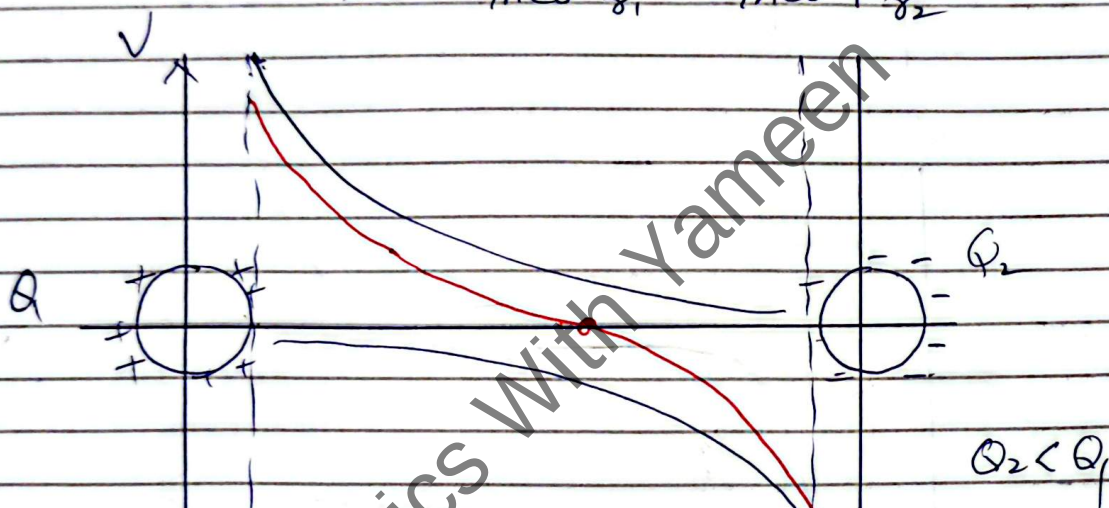
*



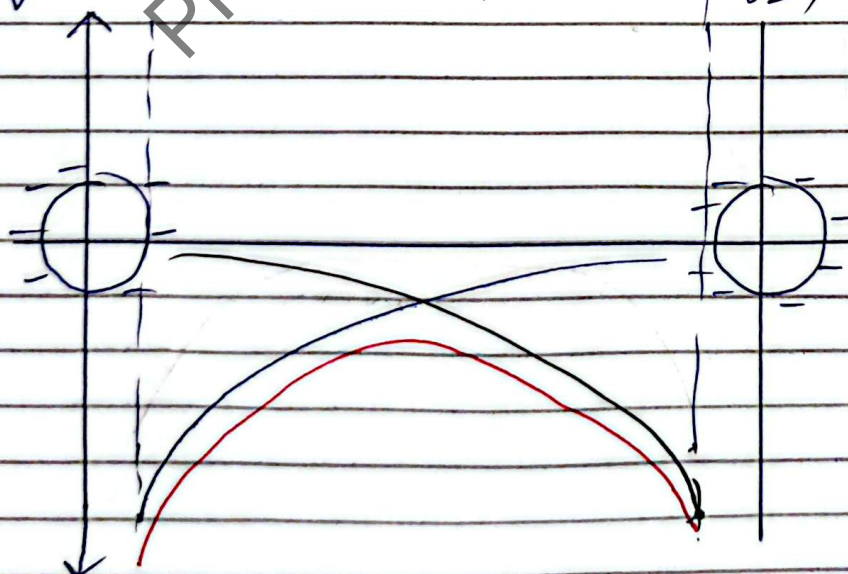
$$Q_1 = Q_2$$



$$V_A = \frac{1}{4\pi\epsilon_0} \frac{Q_1}{r_1} + \frac{1}{4\pi\epsilon_0} \frac{Q_2}{r_2}$$



$$V = \frac{1}{4\pi\epsilon_0} \frac{Q_1}{r_1} + \left(\frac{1}{4\pi\epsilon_0} \frac{(-Q_2)}{r_2} \right)$$



Relation between electric potential and electric field strength.

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \rightarrow (1) \text{ electric field strength}$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r} \rightarrow (2)$$

[This proof is not in the syllabus.]

$$\frac{dv}{dr} = \frac{1}{4\pi\epsilon_0} Q r^{-2}$$

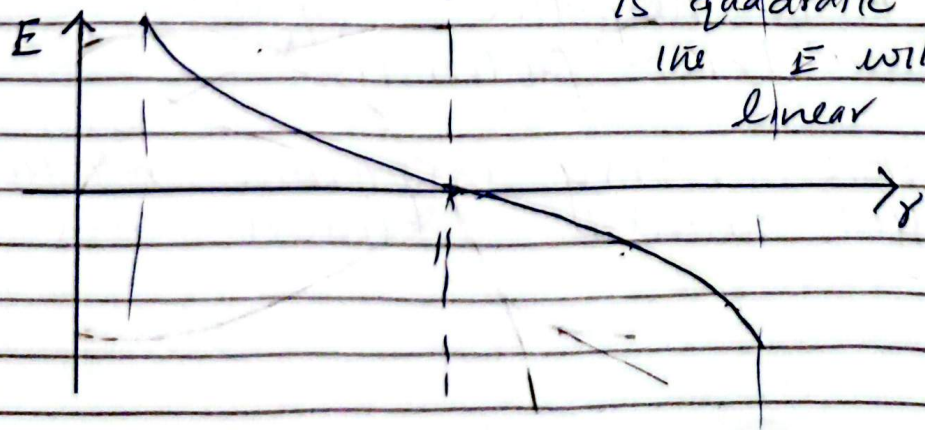
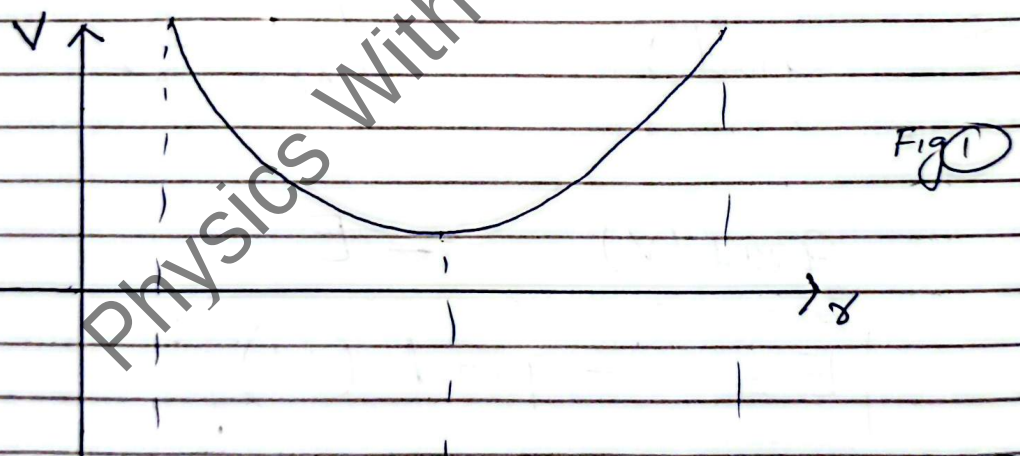
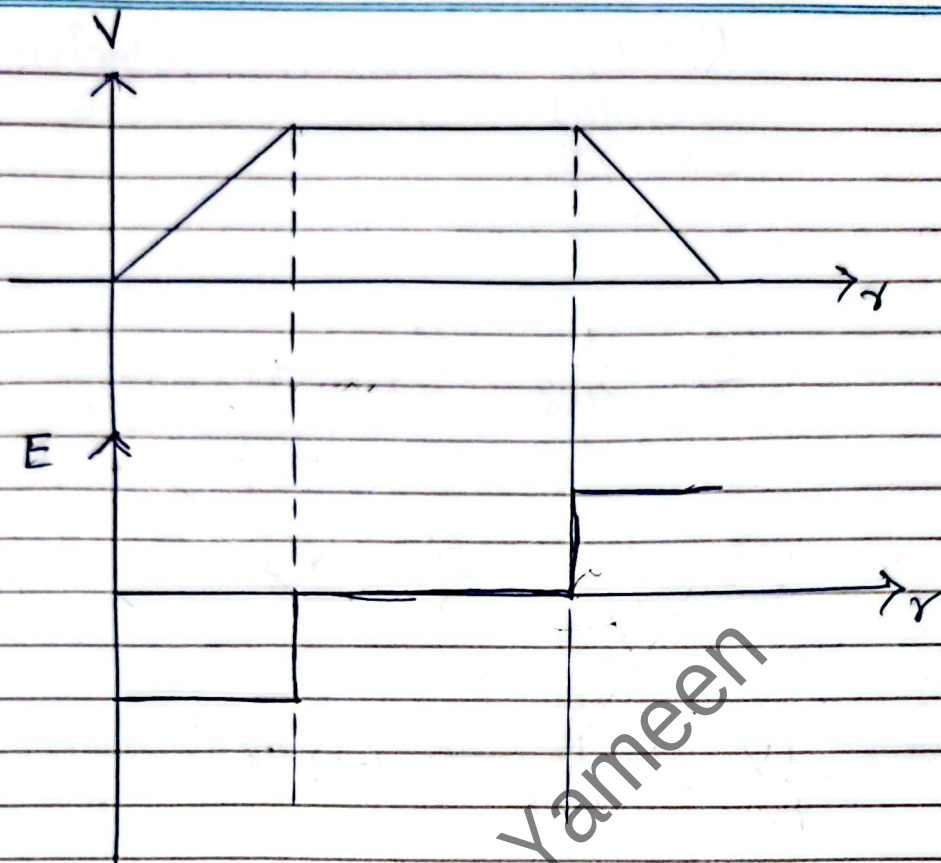
$$\text{grad}(V) = -\frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

$$\text{grad}(V) = -E$$

$$\boxed{E = -\text{grad}(V)}$$

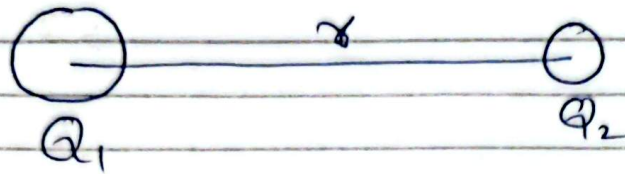
E is negative gradient of electric potential.

This is very important relationship you should know this.



if curve in fig ①
is quadratic
the E will be
linear

Electric potential energy.



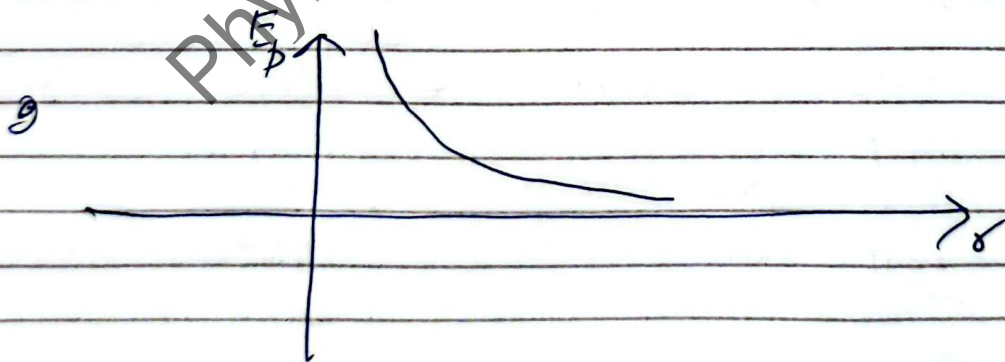
Electric potential energy

$$E_p = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r}$$

* If both charges are like (same sign)

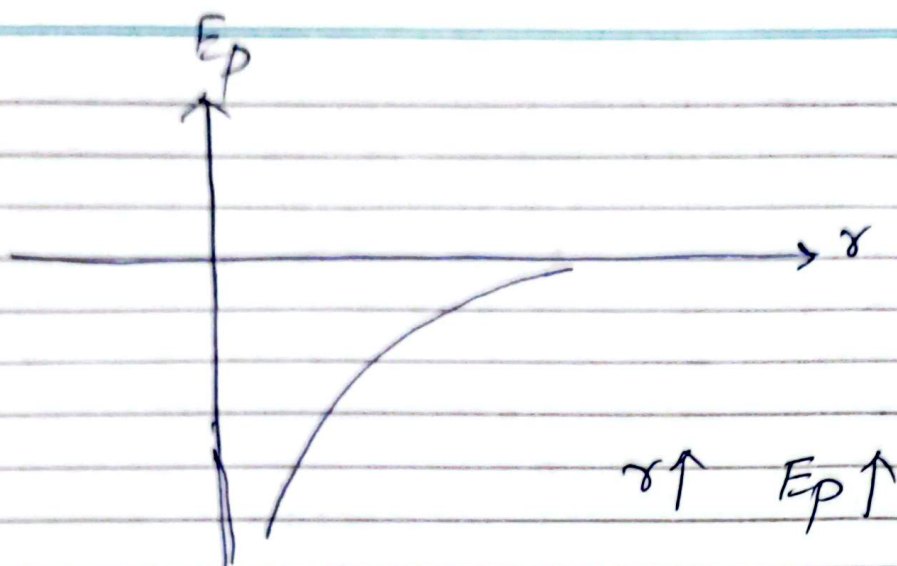
E_p will be positive

E_p will decrease as r is increased.



If both charges are opposite

E_p : negative



(+)

(-)

If two opposite charges are moved in opposite direction then work must be done you need force. Your work done will be used to increase their electric potential energy.

If two opposite charges rest are released from rest

(+)

(-)

(+) $\rightarrow^v$

$\leftarrow^v$ (-)

If their speed is increasing that means E_p should decrease

Electric

Relation between potential energy and electric potential

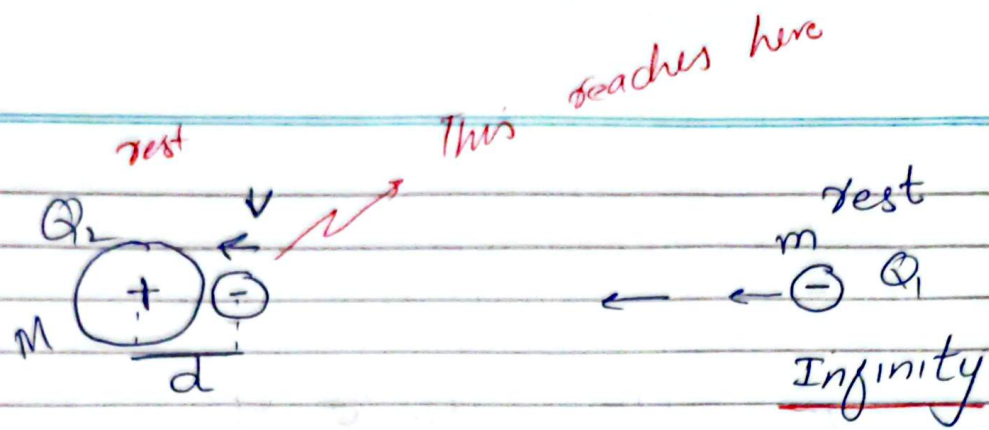
$$E_p = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r}$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

$$E_p = V q$$

$$\Delta E_p = \Delta V q$$

Physics With Yameen



charge Q_1 is released from rest from infinity

find its speed when it touches Q_2 .

W.S.M
P.E = E_i + K.E

$$\frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r} = \frac{1}{2} m v^2$$

d : distance between their centres when they touch.

total energy of the system before and after is same.

Before = After

$$E_p + E_k = E_p + E_k$$

$$0 + 0 = -\frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{d} + \frac{1}{2} m v^2$$

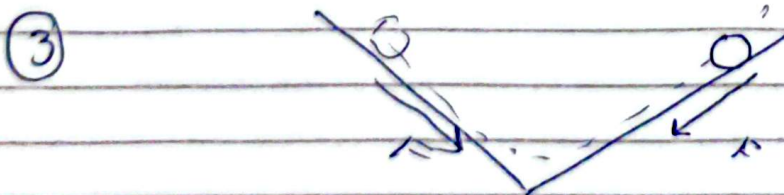
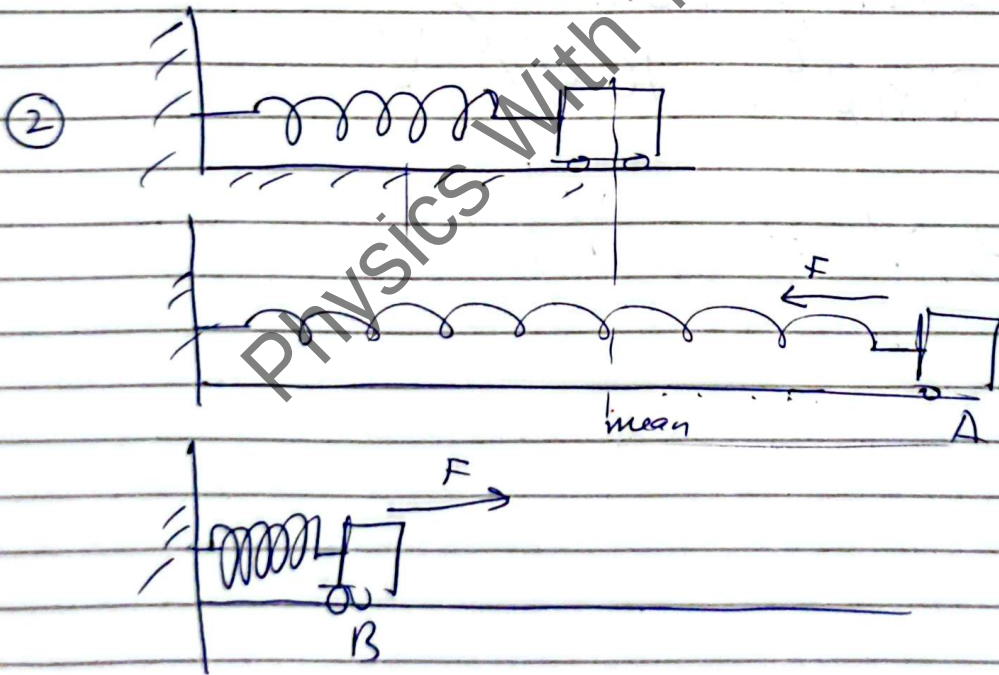
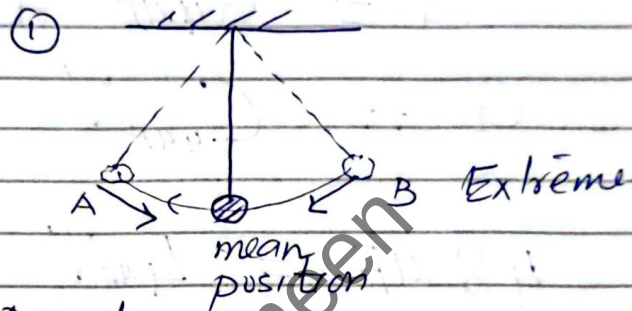
$$\frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{d} = \frac{1}{2} m v^2$$

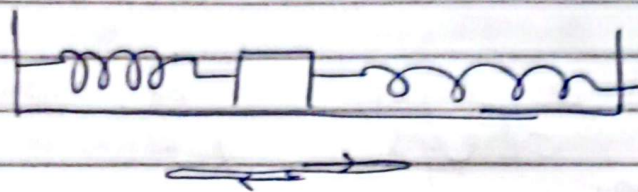
Oscillation

Oscillation

"periodic motion between two ~~fixed~~ points"

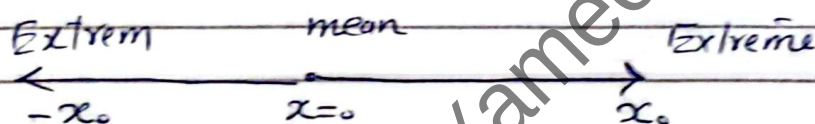
Example



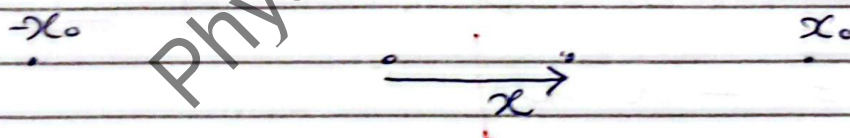


Amplitude

"Maximum displacement from mean position"

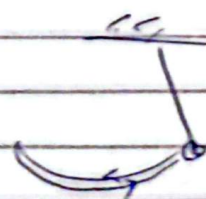


Displacement: shortest distance from mean position



Time period

time to complete one oscillation



Frequency: Number of cycles (oscillations) per unit time.

$$f = \frac{1}{T}$$

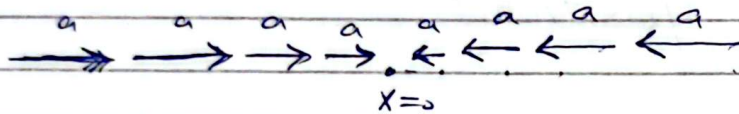
$$\omega = \frac{2\pi}{T}$$

$$\boxed{\omega = 2\pi f}$$

Simple Harmonic Motion (SHM)

this is a type of oscillation which obeys following conditions

- i) Acceleration is directly proportional to displacement from mean position



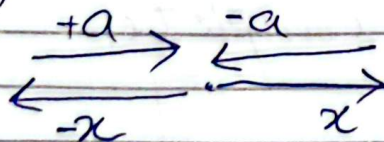
$$a \propto x$$

- ii) acceleration is directed towards the mean position

General Equation of SHM

$$\boxed{a = -\omega^2 x}$$

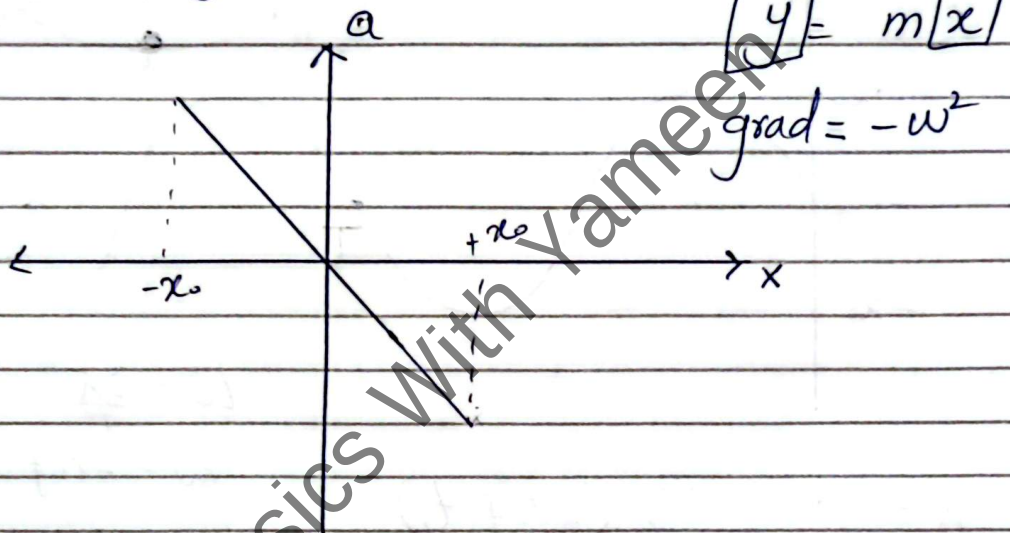
- (-) sign shows a is opposite to displacement



Define SHM?

✓
Motion in which acceleration is directly proportional to displacement from mean position and directed towards mean position

Graph of a and x



Maximum acceleration

$$a_{\max} = -\omega^2 x_0$$

when x is max

Equations for displacement

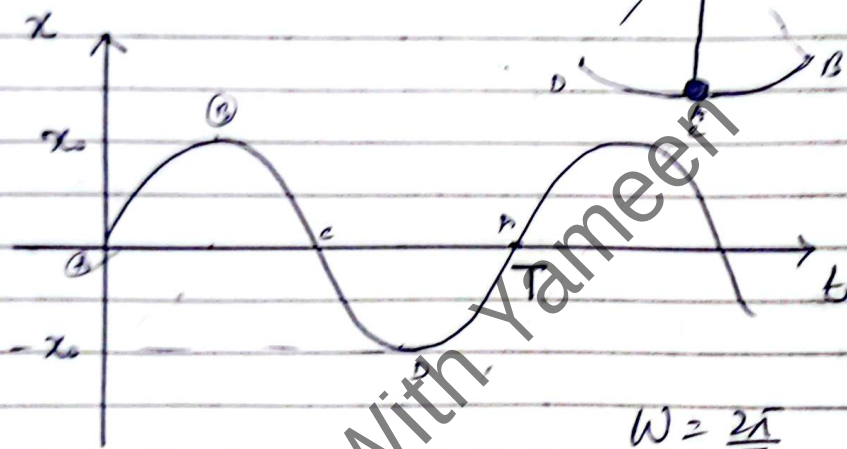
$$x = x_0 \sin(\omega t)$$

this equation satisfies $a = -\omega^2 x$

This is applicable when

object is at mean position

at $t=0$



$$x = x_0 \sin(2\pi f t)$$

$$\omega = \frac{2\pi}{T}$$
$$\omega = 2\pi f$$

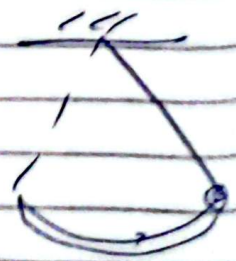
$$x = x_0 \sin\left(\frac{2\pi}{T} t\right)$$

unit of t and T same

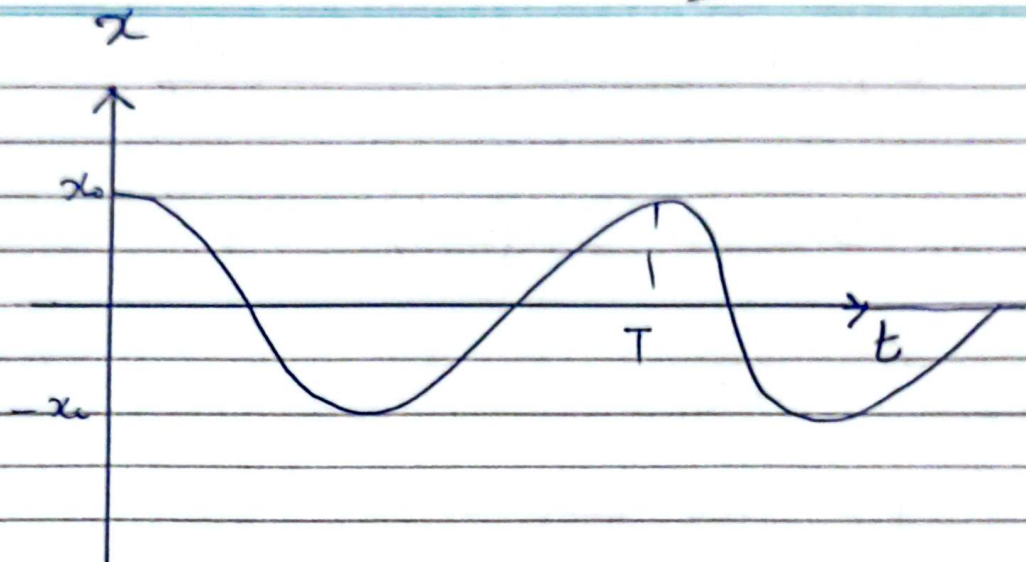
other Equation

$$x = x_0 \cos(\omega t)$$

at $t=0$ $x = x_0$



NY 94 11/11/14



an object starts from mean position at $t=0$ it has amplitude of 10 cm and time period of 0.4 s

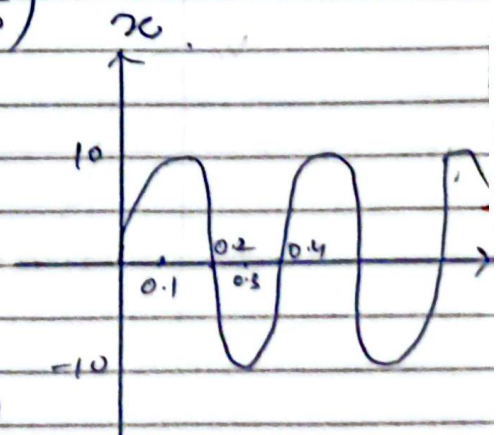
Write down equation for displacement

$$x = x_0 \sin \omega t$$

$$x = 10 \sin \left(\frac{2\pi}{T} t \right)$$

$$x = 10 \sin \left(\frac{2\pi}{0.4} t \right)$$

$$x = 10 \sin (5\pi t)$$



at $t=0$	$x=0$
$t = 0.1$	$x = 10 \text{ cm}$
0.2	$x = 0$
0.3	$x = -10$
0.4	$x = 0$

Equations for Velocity

$$x = x_0 \sin(\omega t)$$

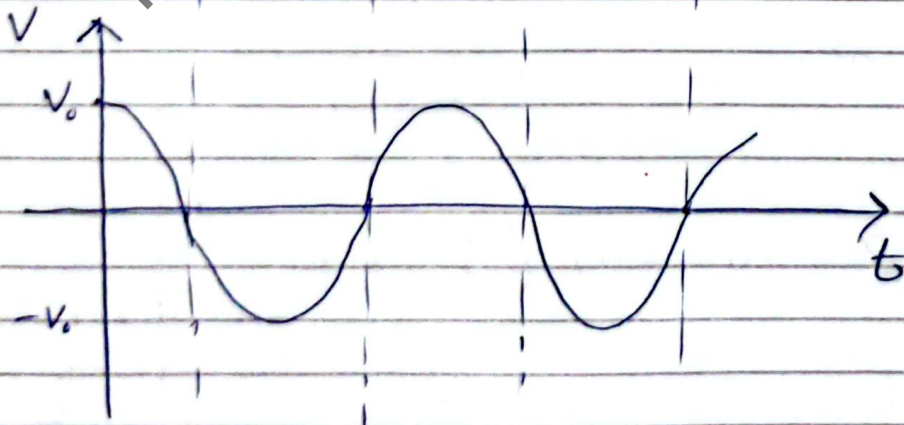
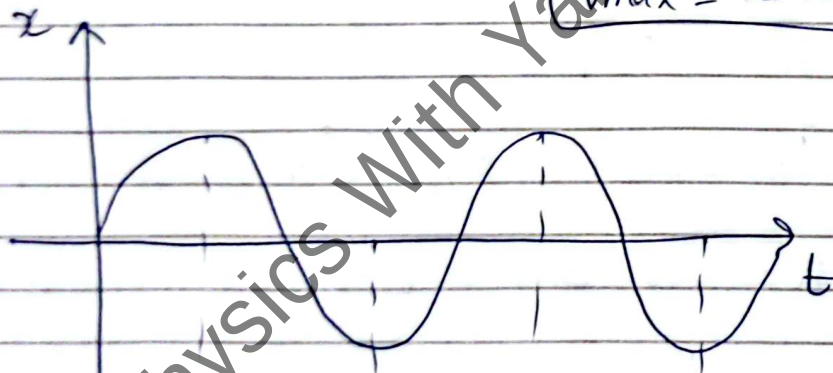
$$V = \frac{dx}{dt}$$

$$V = x_0 \cos(\omega t) \cdot \omega$$

$$V = x_0 \omega \cos(\omega t)$$

$$V = V_0 \cos(\omega t)$$

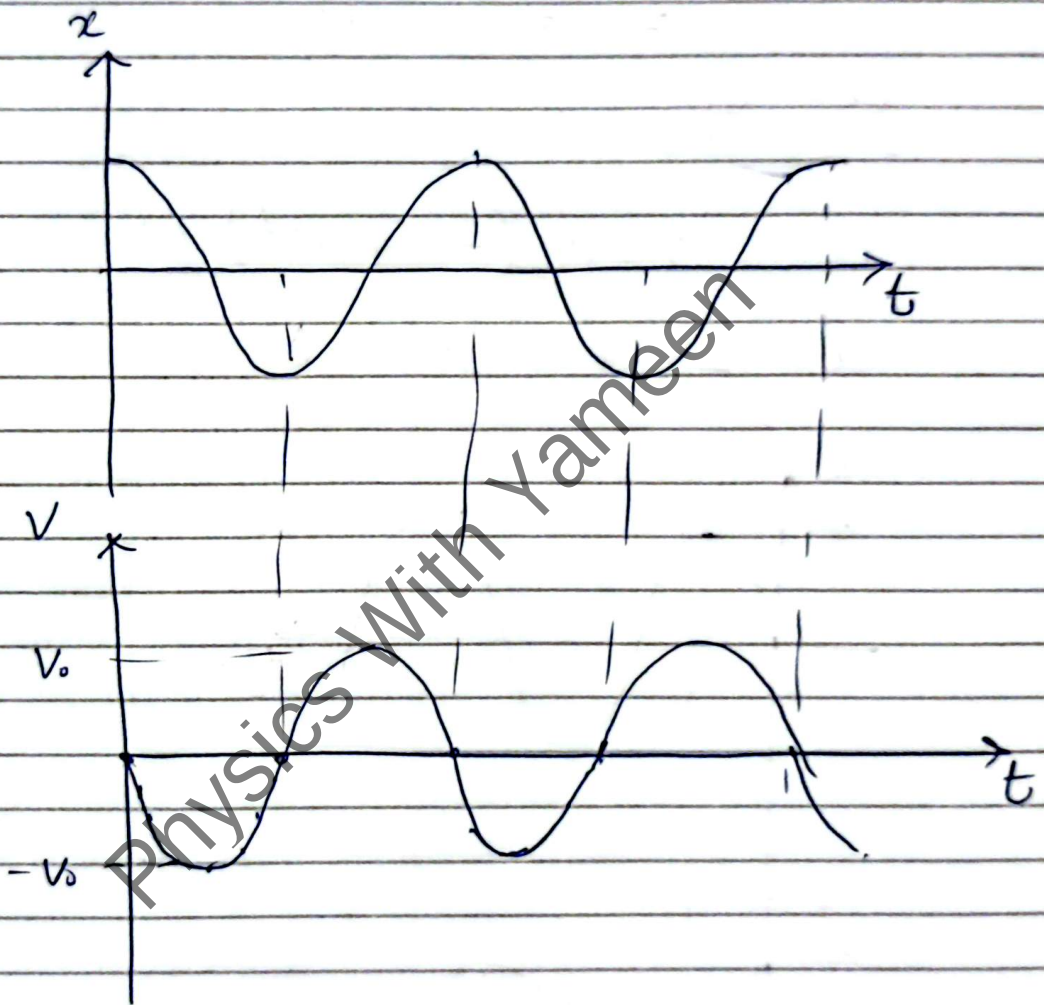
$$V_{\max} = x_0 \omega$$



$$x = x_0 \cos \omega t$$

$$V = -x_0 \omega \sin \omega t$$

$$\frac{dx}{dt} = -x_0 \omega \sin \omega t = V$$



Equations for acceleration

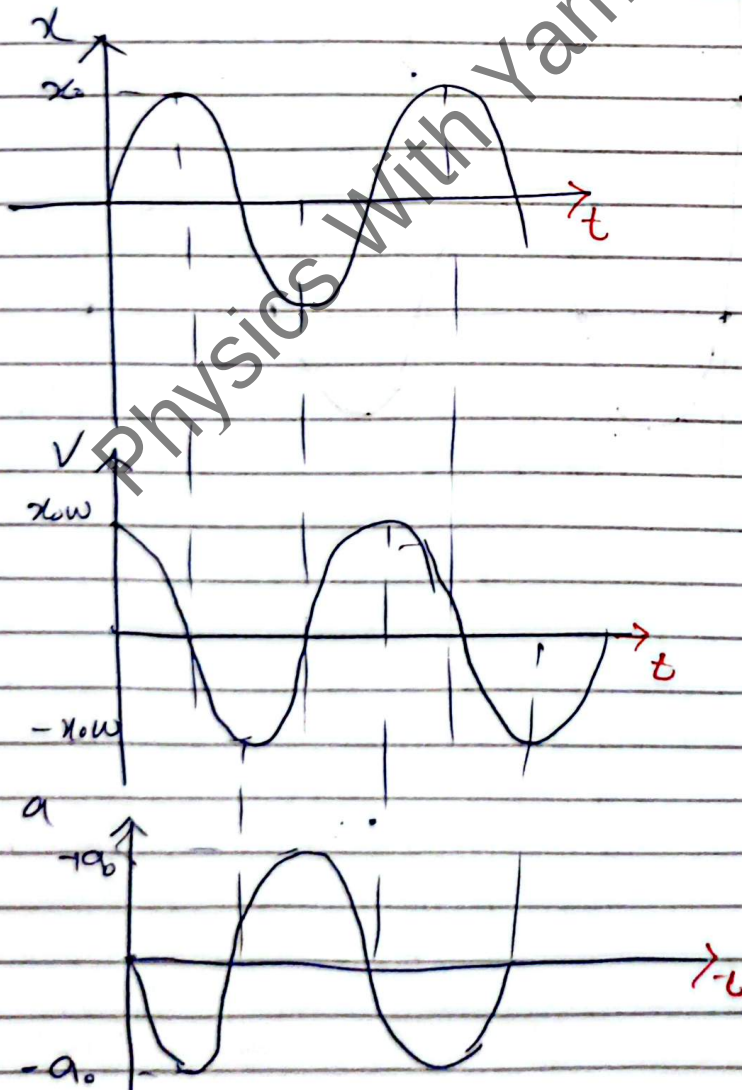
$$x = x_0 \sin \omega t$$

$$v = \underline{x_0 \omega} \cos \omega t$$

$$a = \frac{dv}{dt}$$

$$a = x_0 \omega (-\sin \omega t) \cdot \omega$$

$$a = -x_0 \omega^2 \sin \omega t$$



Equation of velocity with respect to displacement.

$$x = x_0 \sin \omega t \rightarrow (1)$$

$$V = x_0 \omega \cos \omega t \rightarrow (2)$$

from (1)

$$\frac{x}{x_0} = \sin(\omega t)$$

from (2)

$$\frac{V}{x_0 \omega} = \cos(\omega t)$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sin^2 \omega t + \cos^2 \omega t = 1$$

$$\left(\frac{x}{x_0}\right)^2 + \left(\frac{V}{x_0 \omega}\right)^2 = 1$$

$$\frac{x^2}{x_0^2} + \frac{V^2}{x_0^2 \omega^2} = 1$$

$$\frac{\omega^2 x^2 + V^2}{x_0^2 \omega^2} = 1$$

$$\omega^2 x^2 + V^2 = x_0^2 \omega^2$$

$$V^2 = x_0^2 \omega^2 - \omega^2 x^2$$

$$V^2 = x_0^2 \omega^2 - \omega^2 x^2$$

$$V = \pm \sqrt{x_0^2 \omega^2 - \omega^2 x^2}$$

$$V = \pm \sqrt{\omega^2 (x_0^2 - x^2)}$$

$$V = \pm \omega \sqrt{x_0^2 - x^2}$$

Derivation is not in the syllabus

at $x=0$ $\longrightarrow$ mean position

$$V = \pm \omega \sqrt{x_0^2 - 0^2}$$

$$V_{\max} = \pm \omega x_0$$

at $x = x_0$

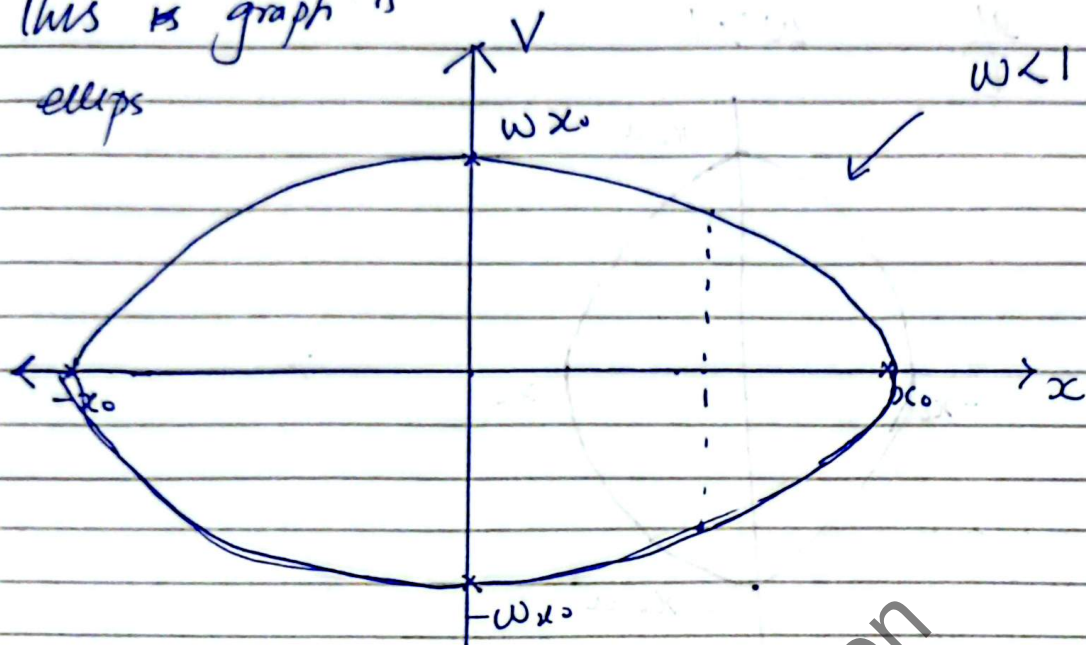
$$V = 0$$

$\downarrow$
extreme point

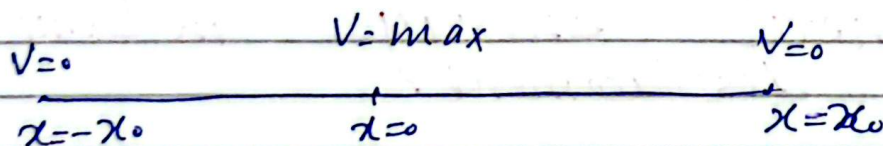
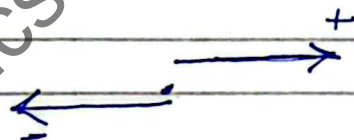
g/ $x_0 = 4 \text{ m}$
 $\omega = 5$

$$V = \pm 5 \sqrt{16 - x^2}$$

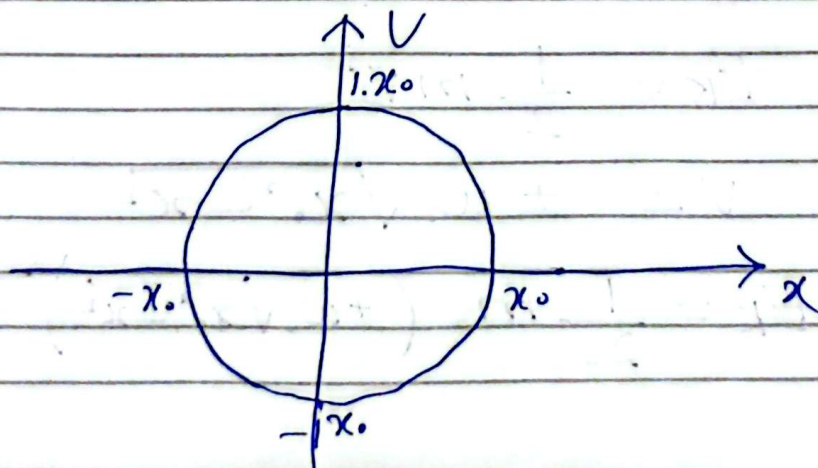
This is graph "is
ellips

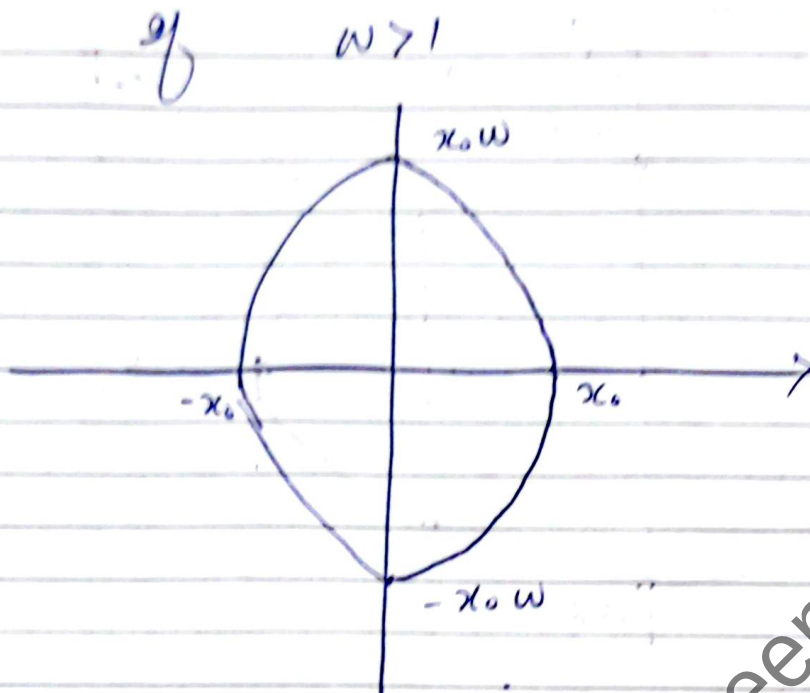


At any value of x there are
two values of V one
is positive and other is
negative



if $\omega = 1$





Kinetic energy in terms
of displacement

$$E_k = \frac{1}{2} m v^2$$

$$v = \pm \omega \sqrt{x_0^2 - x^2}$$

$$E_k = \frac{1}{2} m (\pm \omega \sqrt{x_0^2 - x^2})^2$$

$$E_K = \frac{1}{2} m \omega^2 (x_0^2 - x^2)$$

at $x=0$

$$E_K = \frac{1}{2} m \omega^2 x_0^2$$

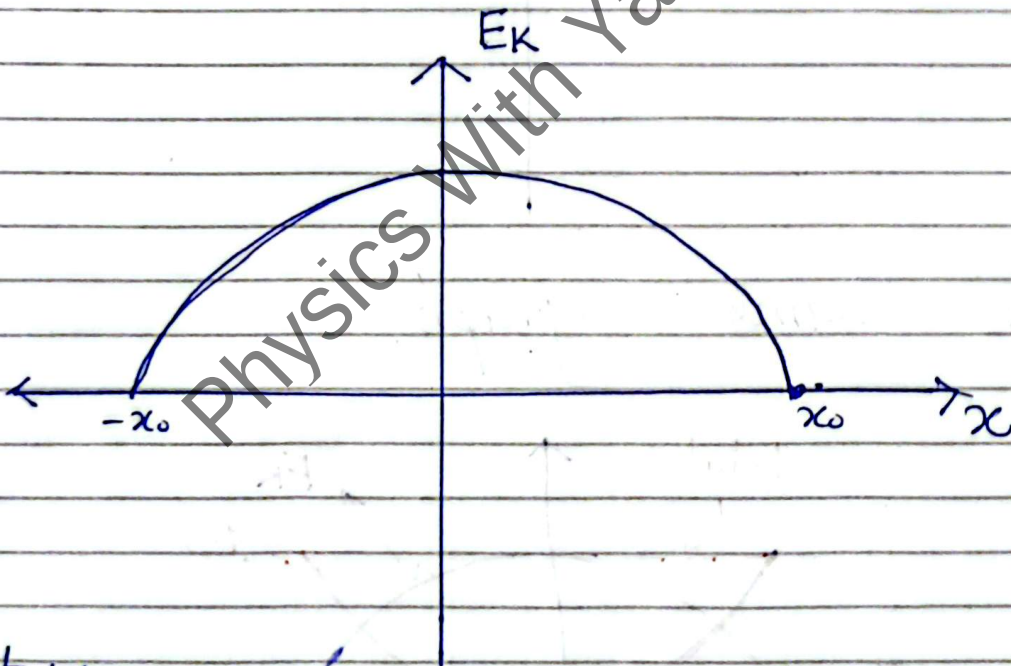
thus it's
max

this is total energy

at $x=x_0$

$$E_K = \frac{1}{2} m \omega^2 (x_0^2 - x_0^2)$$

$$\boxed{E_K = 0}$$



total energy:

which is total energy

at mean K.E is max

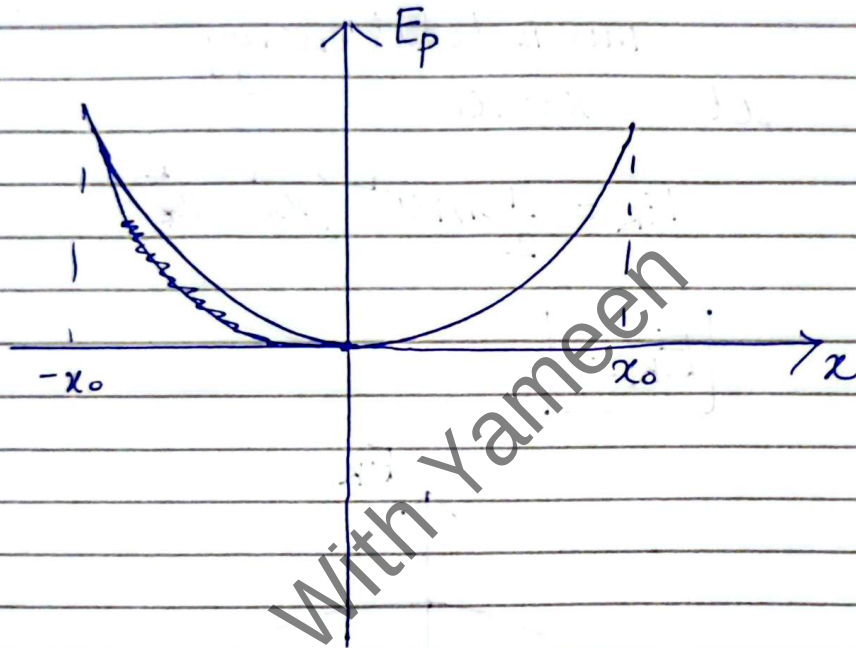
$$E_{Tot} = \frac{1}{2} m \omega^2 x_0^2$$

$$E_p = E_{tot} - E_K$$

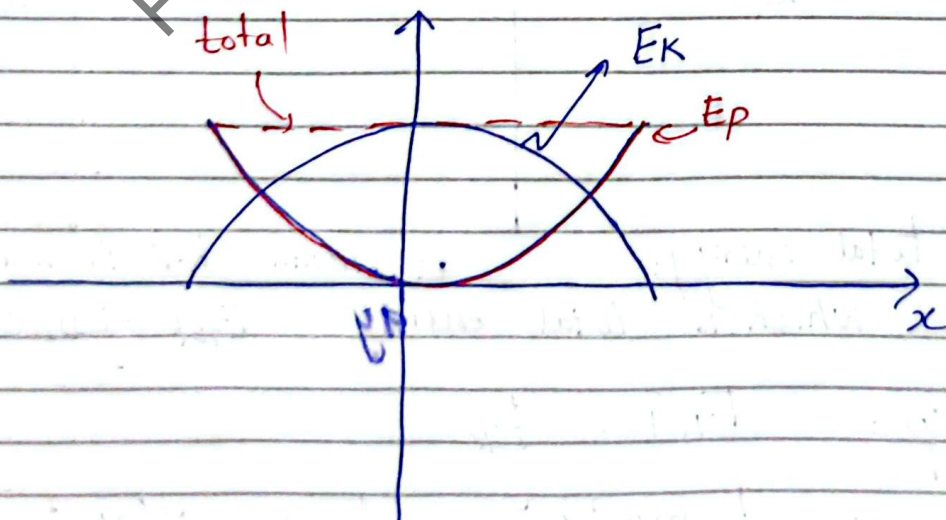
$$= \frac{1}{2} m \omega^2 x_0^2 - \frac{1}{2} m \omega^2 (x_0^2 - x^2)$$

$$E_p = \frac{1}{2} m \omega^2 x_0^2 - \frac{1}{2} m \omega^2 x_0^2 + \frac{1}{2} m \omega^2 x^2$$

$$E_p = \frac{1}{2} m \omega^2 x^2$$



on same graph



Energy variation with respect to time.

$$x = x_0 \sin \omega t$$

$$v = x_0 \omega \cos \omega t$$

$$E_k = \frac{1}{2} m v^2$$

$$E_k = \frac{1}{2} m (x_0 \omega \cos \omega t)^2$$

$$E_k = \frac{1}{2} m \omega^2 x_0^2 \cos^2 \omega t$$

at $t=0$

$$\cos 0 = 1$$

$$E_k = \frac{1}{2} m \omega^2 x_0^2$$

Potential energy

$$E_p = \frac{1}{2} m \omega^2 x^2$$

$$= \frac{1}{2} m \omega^2 (x_0 \sin \omega t)^2$$

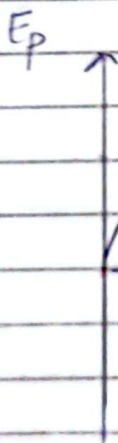
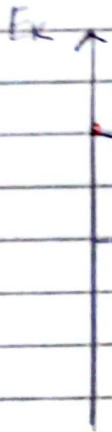
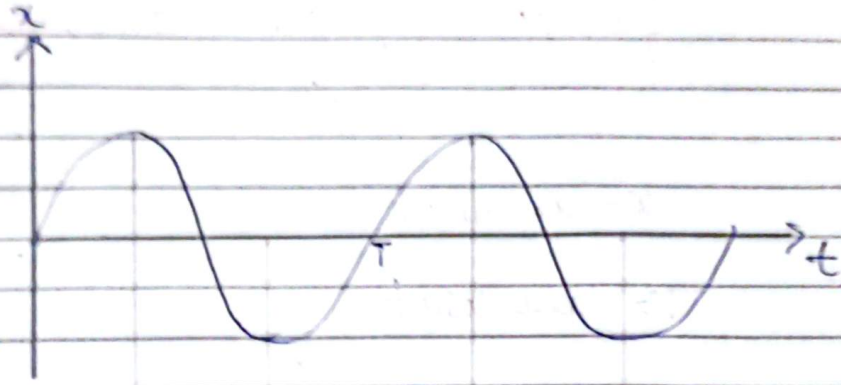
$$E_p = \frac{1}{2} m \omega^2 x_0^2 \sin^2 \omega t$$

$$E_{tot} = E_k + E_p$$

$$= \frac{1}{2} m \omega^2 x_0^2 \cos^2 \omega t + \frac{1}{2} m \omega^2 x_0^2 \sin^2 \omega t$$

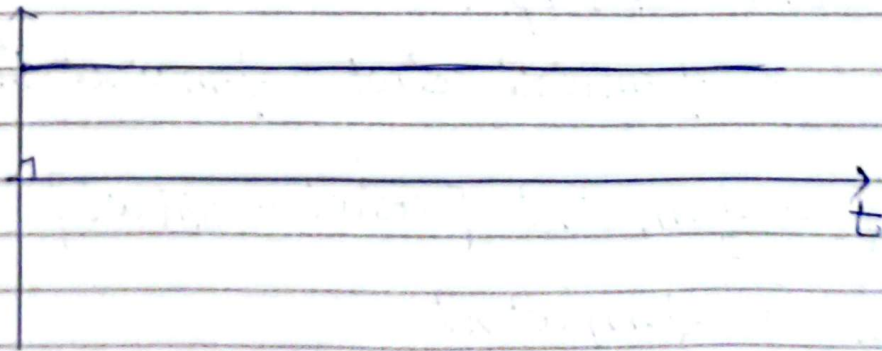
$$E_{tot} = \frac{1}{2} m \omega^2 x_0^2 [\cos^2 \omega t + \sin^2 \omega t]$$

$$E_{tot} = \frac{1}{2} m \omega^2 x_0^2$$



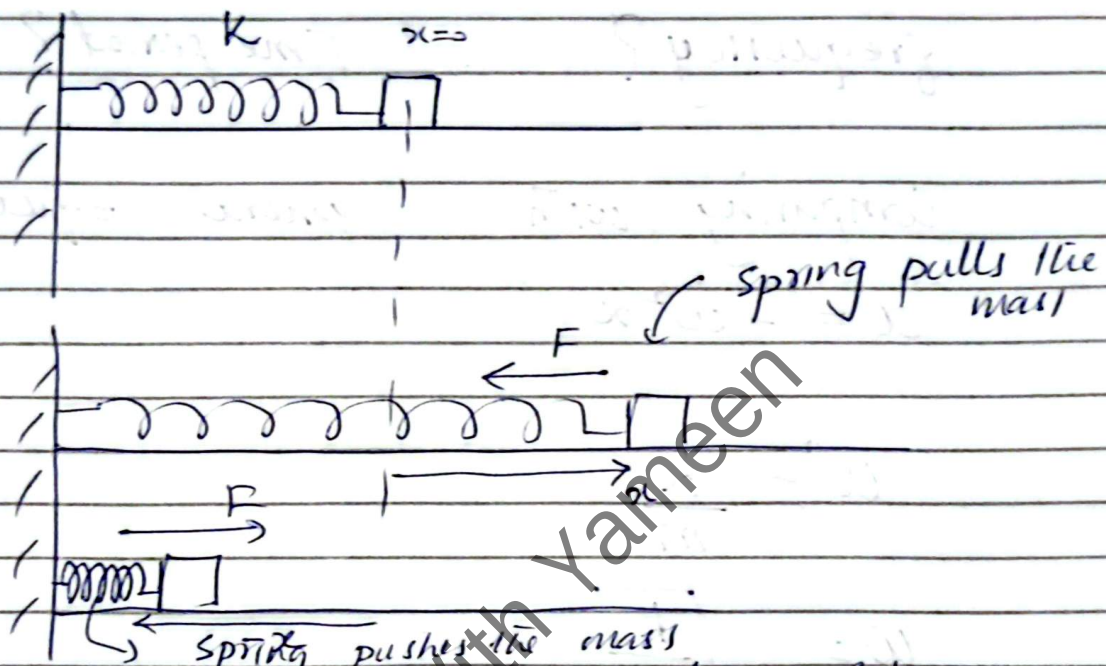
$$\frac{1}{2} m \omega^2 x^2 \sin^2 \omega t$$

E_{tot}



Physics With Yameen

Examples of Simple Harmonic motion.



Mass is pulled towards right by some displacement x and then released

Tension of the spring acts in back ward direction

$$F = -kx$$

Hooke's law

$$ma = -kx$$

$$a = -\frac{k}{m}x$$

k and m are constant so

$$a \propto x$$

(-) shows a is towards mean

$$a = -\frac{k}{m}x$$

So it is simple harmonic motion

frequency?

Time period?

Comparing with general equation

$$a = -\omega^2 x$$

$$\omega^2 = \frac{k}{m}$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{\sqrt{\frac{k}{m}}}$$

$$T = 2\pi \sqrt{\frac{m}{k}}$$

$$f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$\boxed{T = W}$$

$$ke = W$$

In c Tension is greater than Weight



↓ x (+)

$$F_{\text{net}} = -T + W$$

$$F = -k(x+e) + W$$

$$F_{\text{net}} = -kx - ke + W$$

as $ke = W$

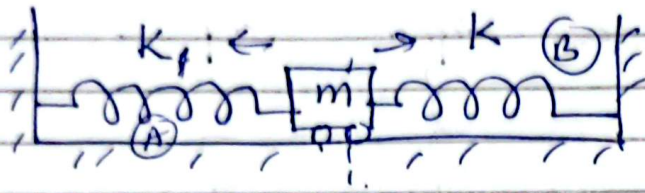
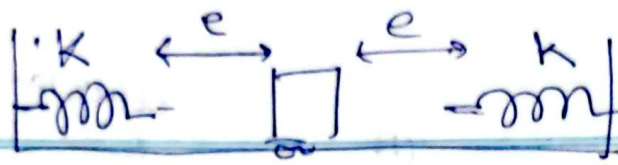
$$F_{\text{net}}$$

$$F = -kx$$

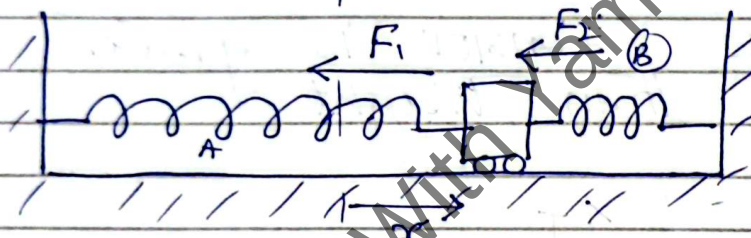
$$ma = -kx$$

$$a = -\frac{k}{m}x$$

so SHM



of both springs have $12x$
 already extended by amount 'e'
 When mass m is moved towards
 right by amount of x



Spring A total extension
 $x + e$

$$F_1 = k(x + e)$$

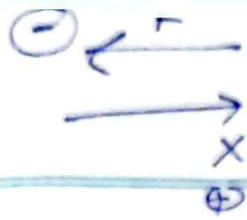
x is greater e

Spring B is compressed Now

total compression of spring B

$$x - e$$

$$F_2 = k(x - e)$$



$\therefore$ sign bes of direction

$$F_{\text{net}} = -F_1 - F_2$$

$$F = -k(x+e) - k(x-e)$$

$$F = -kx - \cancel{k}e - kx + \cancel{k}e$$

$$F = -2kx$$

$$ma = -2kx$$

$$a = -\frac{2k}{m}x \rightarrow (1)$$

as k and m constant

so $a \propto x$

(-) sign shows a is towards equilibrium position

so it performs S.H.M

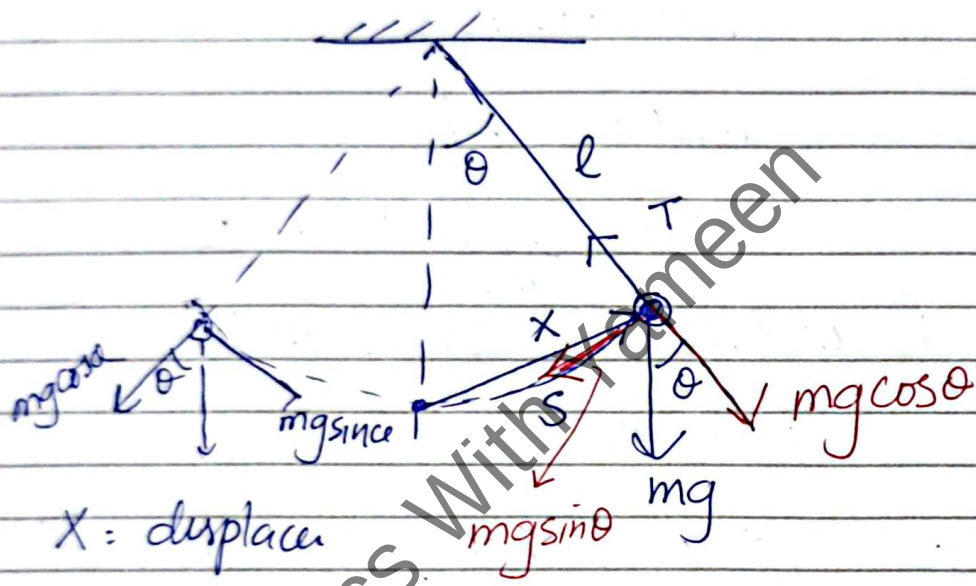
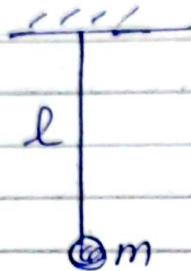
$$a = -\omega^2 x \rightarrow (2)$$

comparing (1) and (2)

$$\omega^2 = \frac{2k}{m}$$

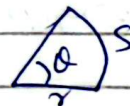
$$\omega = \sqrt{\frac{2k}{m}}$$

Simple pendulum



x : displac

$$s = l\theta$$



θ : radian

$$F = mg \sin \theta$$

when θ is small

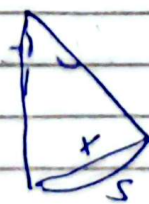
$$\sin \theta \approx \theta$$

$$\sin 0.1 = 0.099$$

$$x \approx s$$

$$F = mg \theta$$

$$\theta = \frac{x}{l}$$



$$F = mg \frac{x}{l}$$

$$F = \frac{mg}{l} x$$

We can introduce (-) sign as
 x and F are in opposite direction

$$F = -\frac{mg}{l} x$$

$$ma = -\frac{mg}{l} x$$

$$a = -\frac{g}{l} x \rightarrow \textcircled{1}$$

g and l are constant

so $a \propto x$

Hence it performs SHM

ω ? T ? f ?

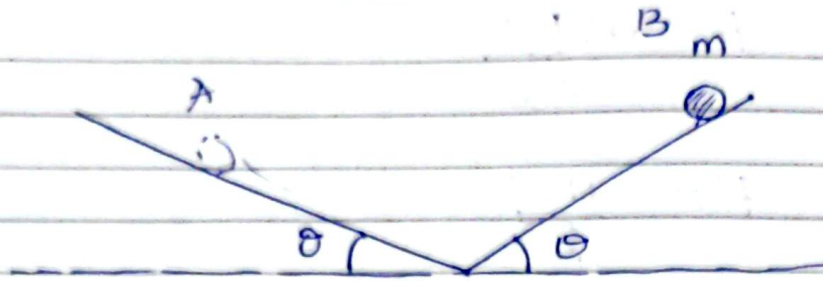
Comparing with $a = -\omega^2 x \rightarrow \textcircled{2}$

$$\omega^2 = \frac{g}{l} \quad \omega = \sqrt{\frac{g}{l}}$$

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{l}{g}}$$

$$T = \frac{2\pi}{\sqrt{g/l}} = 2\pi \sqrt{\frac{l}{g}}$$

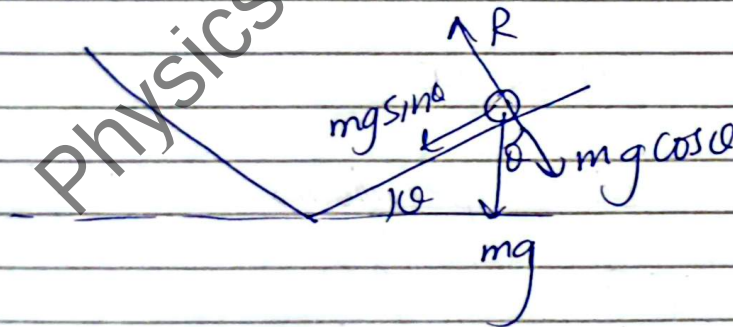
Example which is not SHM



an object of mass m is oscillating on inclined plane b/w A and B . Assume no friction.

Is this a simple Harmonic or not?

For this calculate acceleration



$$F = mg \sin \theta$$

θ : constant

$$ma = mg \sin \theta$$

$$a = g \sin \theta$$

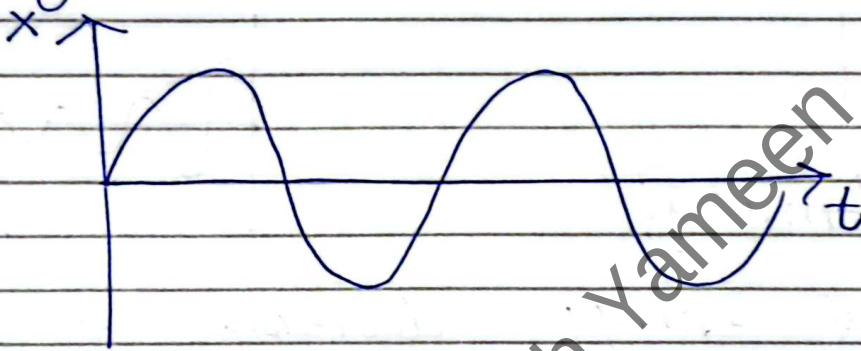
a is constant so
not a SHM

Damping

Free oscillation

No loss of energy due to ex or no gain of energy

of There is no external resultant force.



amplitude should be constant

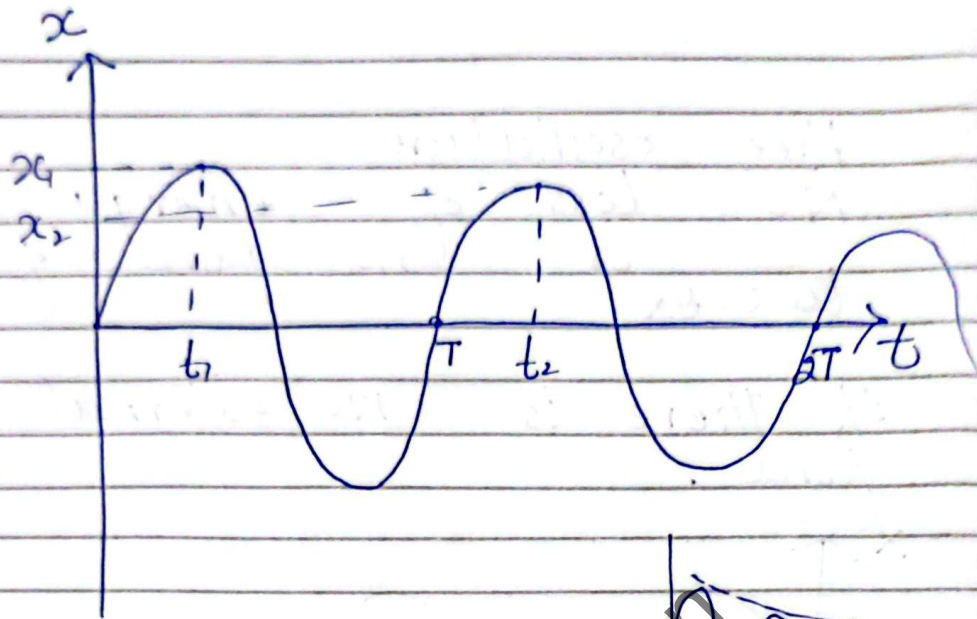
Damped oscillation

"Loss of energy due to external resistive forces"

Types

1) Light damping

oscillation is there but amplitude decreases slowly



$$E_{\text{tot}} = \frac{1}{2} m \omega^2 x_0^2$$

of total energy decreases so amplitude decreases.

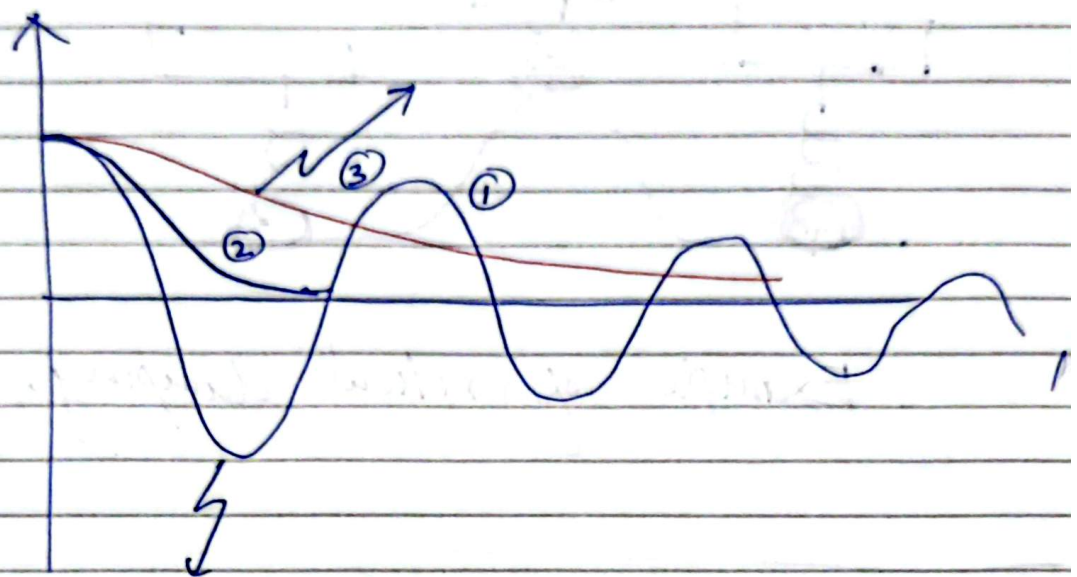
ω and T is constant.

Loss of energy from t_1 to t_2

$$E_{\text{loss}} = \frac{1}{2} m \omega^2 x_1^2 - \frac{1}{2} m \omega^2 x_2^2$$

$$= \frac{1}{2} m \omega^2 (x_1^2 - x_2^2)$$

equation $x = x_0 \sin \omega t$ is not valid. why?



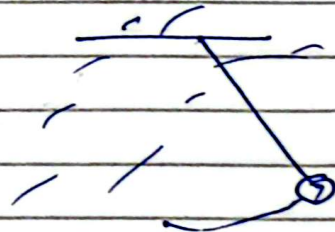
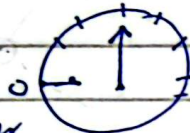
Light damping

2: critical damping

(No oscillation)

Example:

Shock absorber

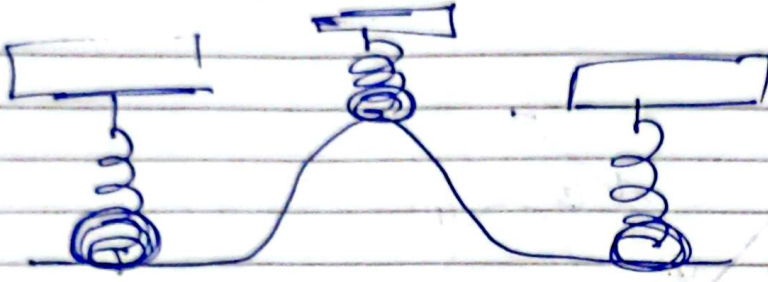


3. over damping
heavy damping

* takes too much time to come to mean position

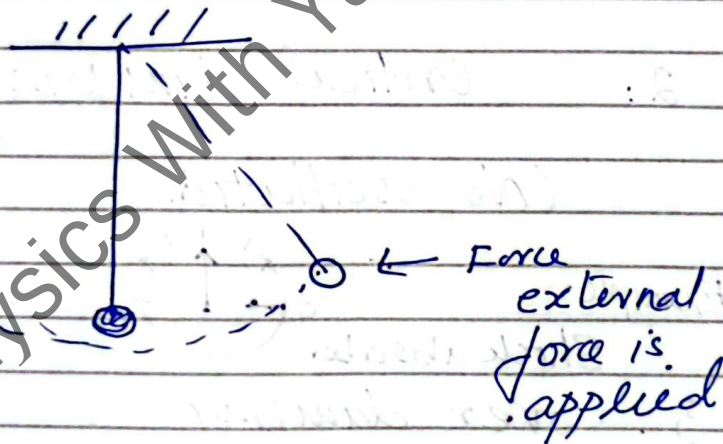
* No oscillation

Shock absorber



Example of critical damping.

Driven System



If energy is supplied (provided) to oscillating body externally by applying periodic force then it becomes driven system

AC source

vibrator
 f : driving frequency

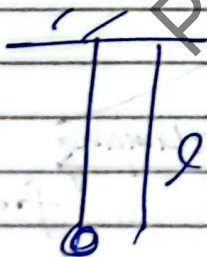
vibrator is off

when vibrator is on m will oscillate with frequency of vibrator

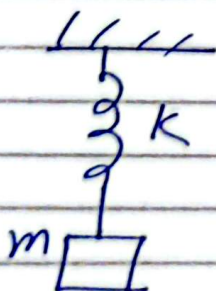
when mass m is pulled down and released it will vibrate (oscillate) with its natural frequency

Natural frequency

frequency with which an object will oscillate if no external net force is applied

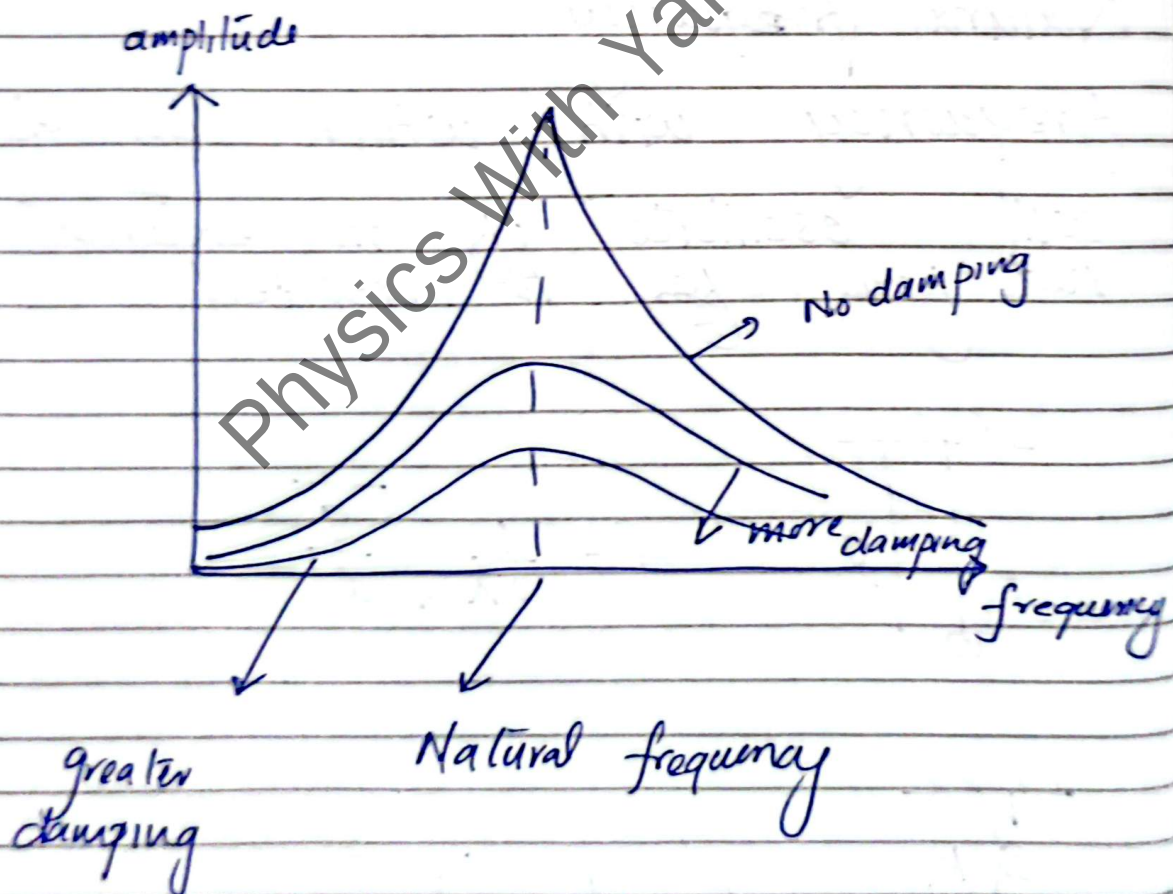


$$f = \frac{1}{2\pi} \sqrt{\frac{g}{l}}$$



$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

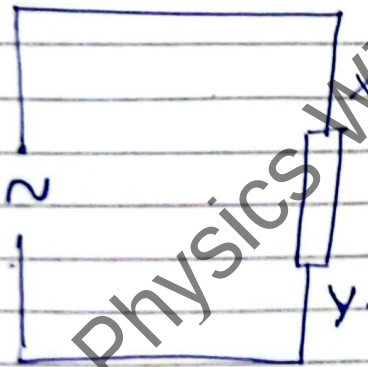
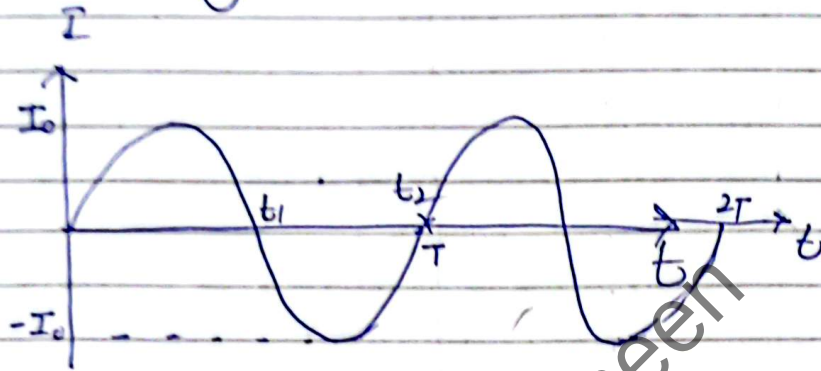
When frequency of vibrator is equal to natural frequency of mass m then the mass starts to vibrate with amplitude of mass m becomes very large and this phenomenon is known as resonance.



AC current

Alternating current (AC)

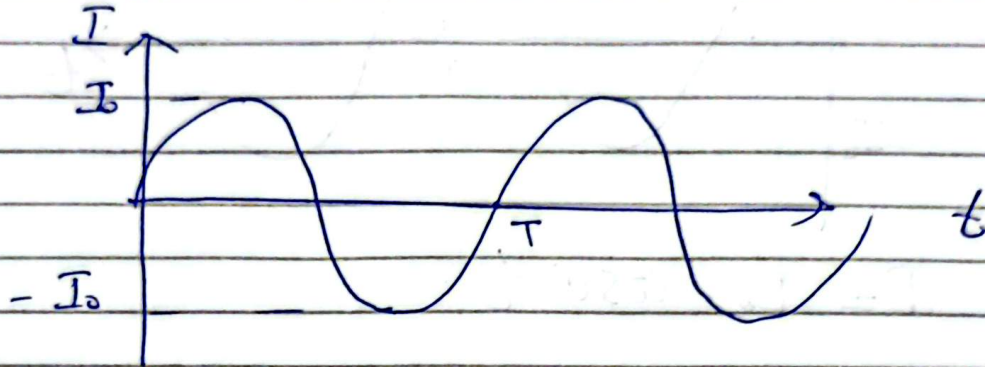
Current which changes direction periodically.



from 0 to t_1 current flows
from x to y and from t_1 to t_2
current flows from y to x
and this process repeats periodically.

other types of AC current

① Sinusoidal current



peak current: Maximum current

Equation

$$I = I_0 \sin(\omega t)$$

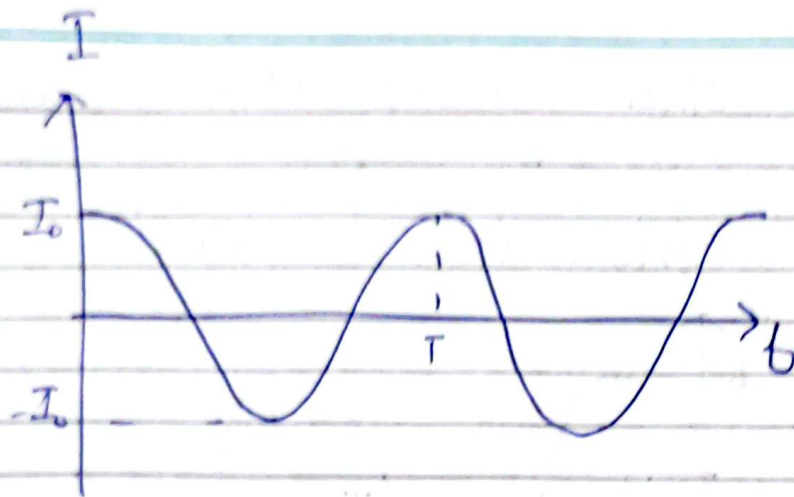
$$\omega = \frac{2\pi}{T} = 2\pi f \quad \omega: \text{angular frequency}$$

$$I = I_0 \sin(2\pi f t)$$

f : frequency

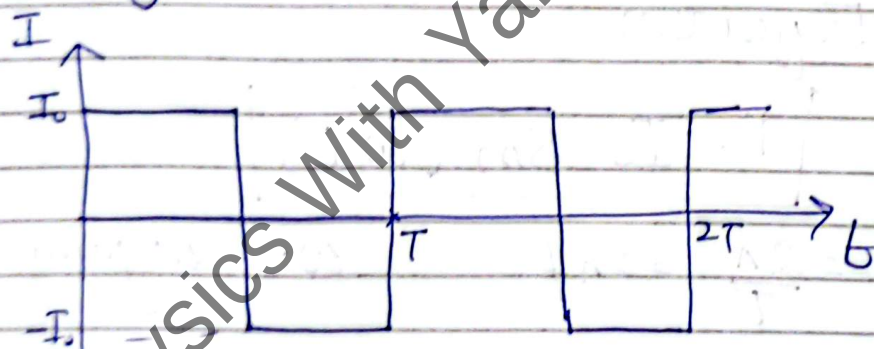
Number of cycles per unit time.

$$T = \frac{1}{f}$$



$$I = I_0 \cos \omega t$$

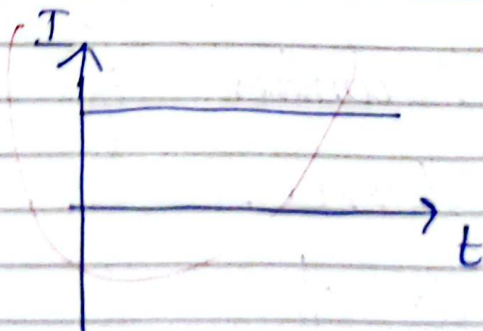
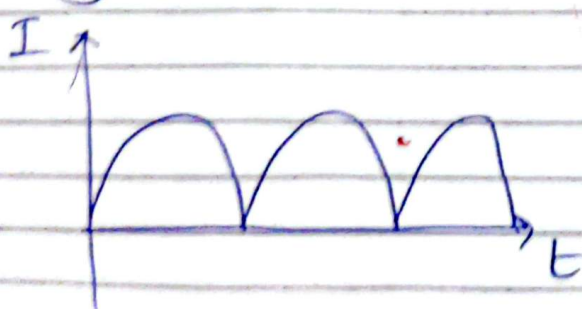
② Rectangular AC current

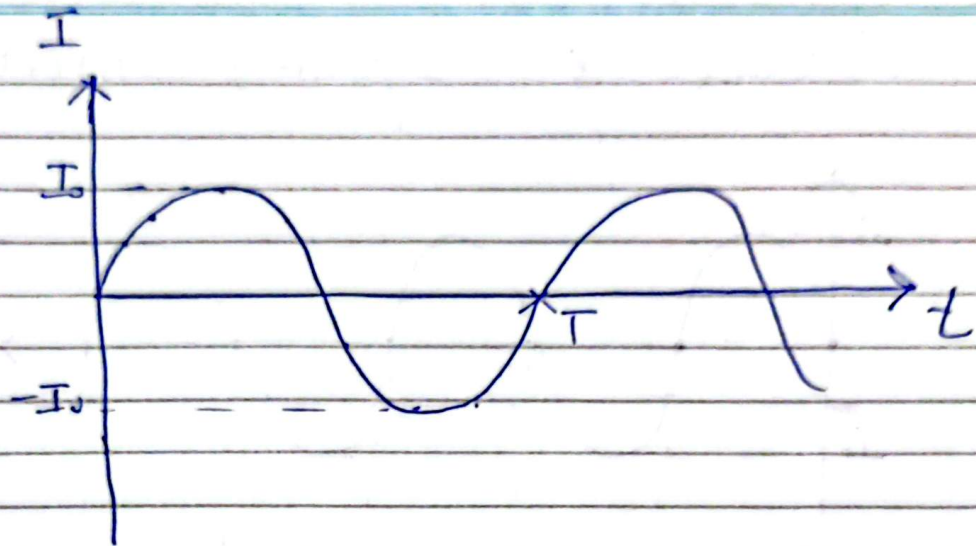


③

R_{ac}

DC current





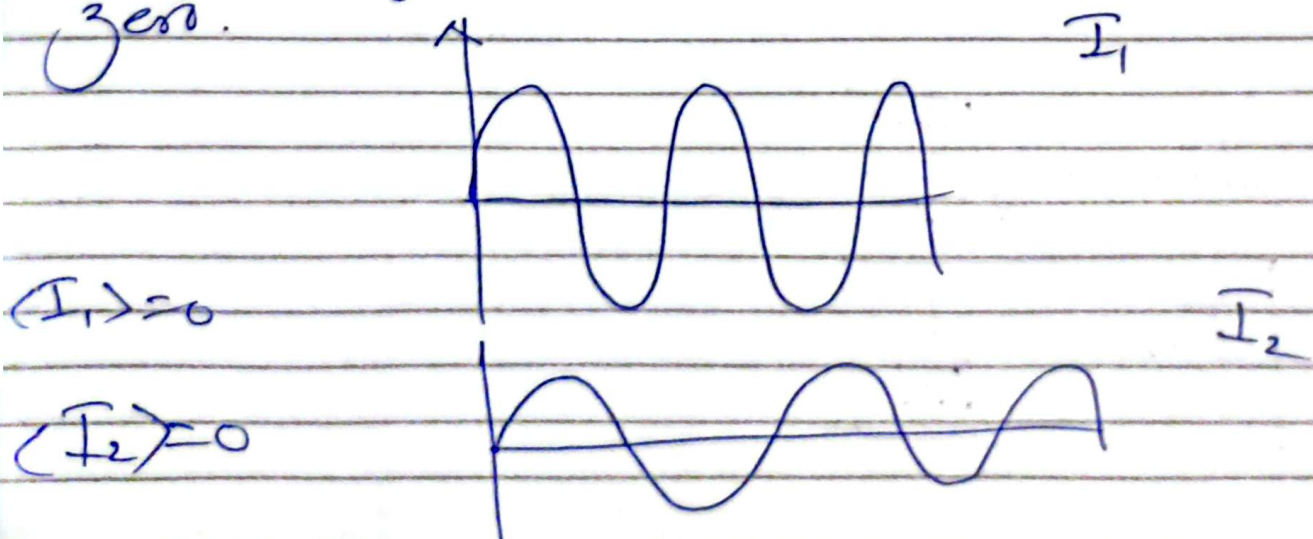
$$I = I_0 \sin \omega t$$

Mean current $\rightarrow \langle I \rangle$

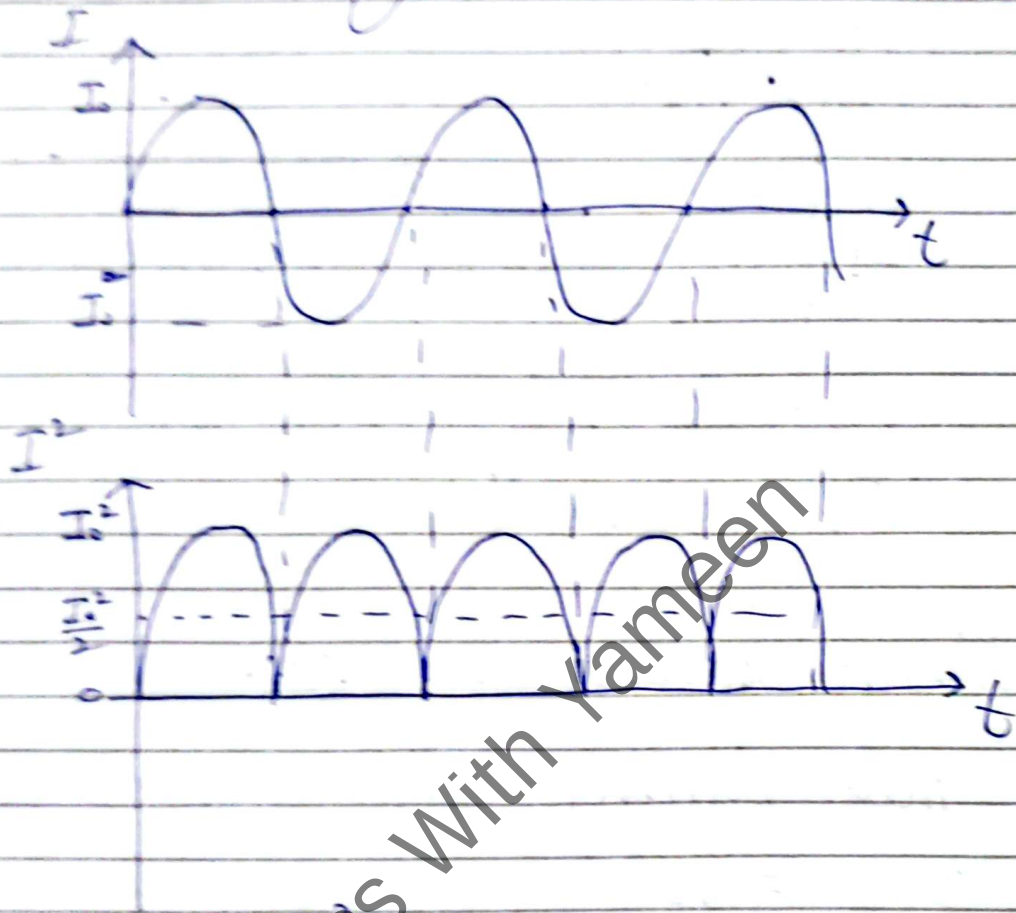
Mean current or average current
is zero

$$\langle I \rangle = 0$$

average of sinusoidal current is
zero.



Average of squared current



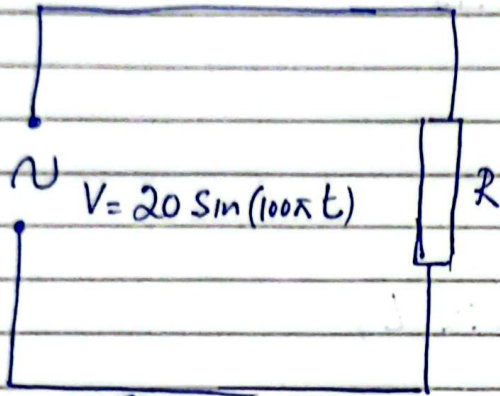
$$\langle I^2 \rangle = \frac{I_0^2}{2}$$

$\sqrt{\langle I^2 \rangle}$ = Root mean square value

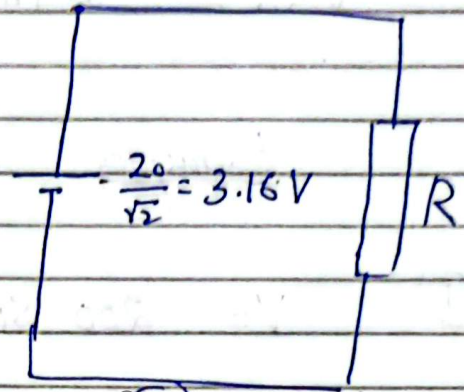
$$I_{rms} = \sqrt{\frac{I_0^2}{2}}$$

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

Definition of root-mean squared value



①

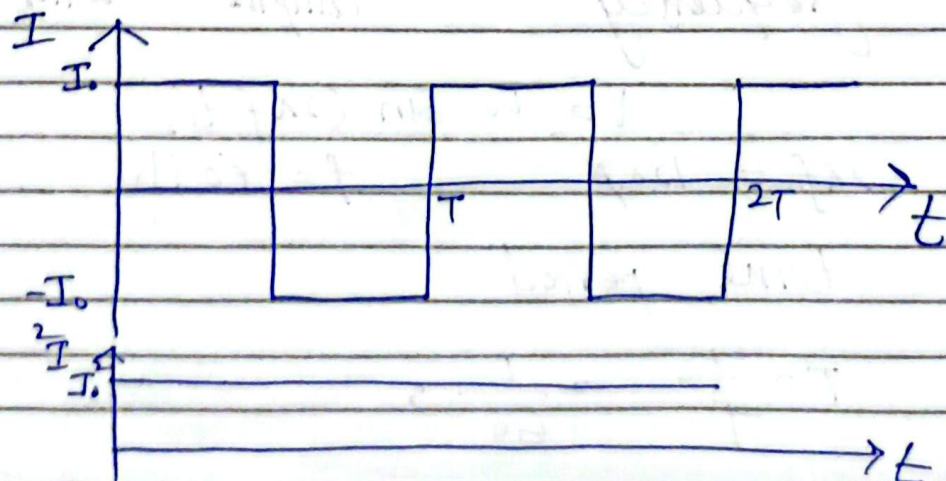


②

It is that value of direct current or voltage that gives same heating effect as AC current gives.

In above example same power is dissipated along both resistors

Rms value of rectangular AC current



$$\langle I^2 \rangle = I_0^2$$

$$\sqrt{\langle I^2 \rangle} = I_{\text{rms}} = I_0$$

Examples

① $V = 200 \sin(120\pi t)$

V is potential difference in volt
 t is in second

Find

i) peak voltage

$$V_0 = V_{\text{peak}} = 200 \text{ V}$$

ii) Average voltage $\langle V \rangle$

$$\langle V \rangle = 0$$

iii) frequency compare with

$$V = V_0 \sin(2\pi f t)$$

$$2\pi f = 120\pi$$

$$f = 60 \text{ Hz}$$

iv) time period

$$T = \frac{1}{f} = \frac{1}{60} \text{ s}$$

Root mean squared voltage

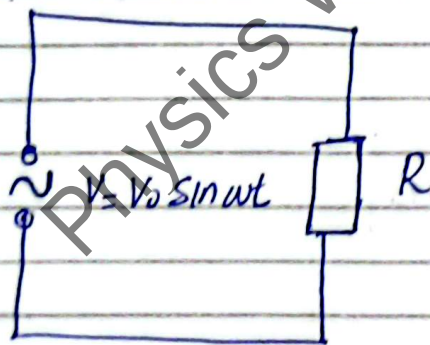
V_{rms} ?

$$V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{200}{\sqrt{2}} = 141.4 \text{ V}$$

Power in AC

$$I = I_0 \sin \omega t$$

$$V = V_0 \sin \omega t$$



$$P = I^2 R$$

$$\sin^2 x_{\max} = 1$$

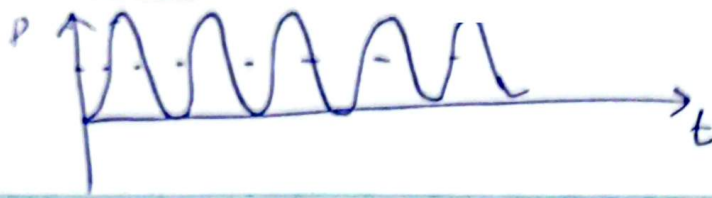
$$P = I_0^2 \sin^2 \omega t R$$

$$\sin^2 x_{(\min)} = 0$$

$$P = I_0^2 R \sin^2(\omega t)$$

$$P_{\max} = I_0^2 R$$

$$P_{\min} = 0$$



Average power

$$\langle P \rangle = \frac{I_0^2 R}{2}$$

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

$$I_{rms}^2 = \frac{I_0^2}{2}$$

$$\boxed{\langle P \rangle = I_{rms}^2 R}$$

$$P = VI$$

$$P = V_0 \sin \omega t \times I_0 \sin \omega t$$

$$P = V_0 I_0 \sin^2(\omega t)$$

$$\langle P \rangle = \frac{V_0 I_0}{2}$$

$$\langle P \rangle = \frac{V_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}}$$

$$\sqrt{2} \times \frac{1}{\sqrt{2}} = 1$$

$$\boxed{\langle P \rangle = V_{rms} \times I_{rms}}$$

$$P = \frac{V^2}{R}$$

$$P = \frac{V_0^2 \sin^2 \omega t}{R}$$

$$\langle P \rangle = \frac{V_0^2}{2R}$$

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$

$$V_{rms}^2 = \frac{V_0^2}{2}$$

$$\boxed{\langle P \rangle = \frac{V_{rms}^2}{R}}$$

Rectification

Process of converting AC to DC

There are two types of rectifications

① ~~Half~~ Half Wave rectification

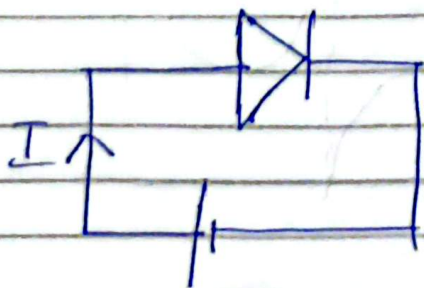
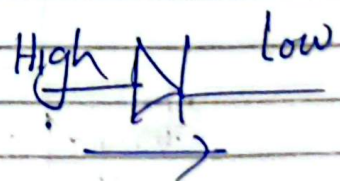
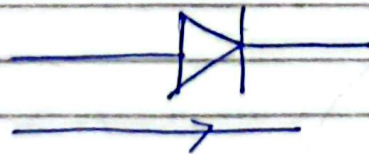
② Full Wave rectification

Half wave

Diode is used to perform rectification.

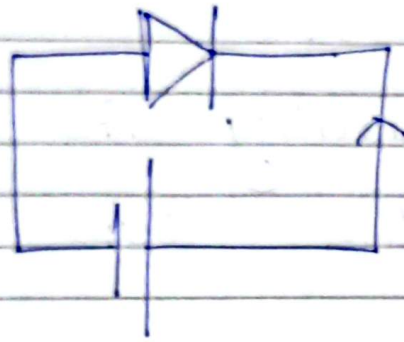
Diode allows current only in direction

Symbol



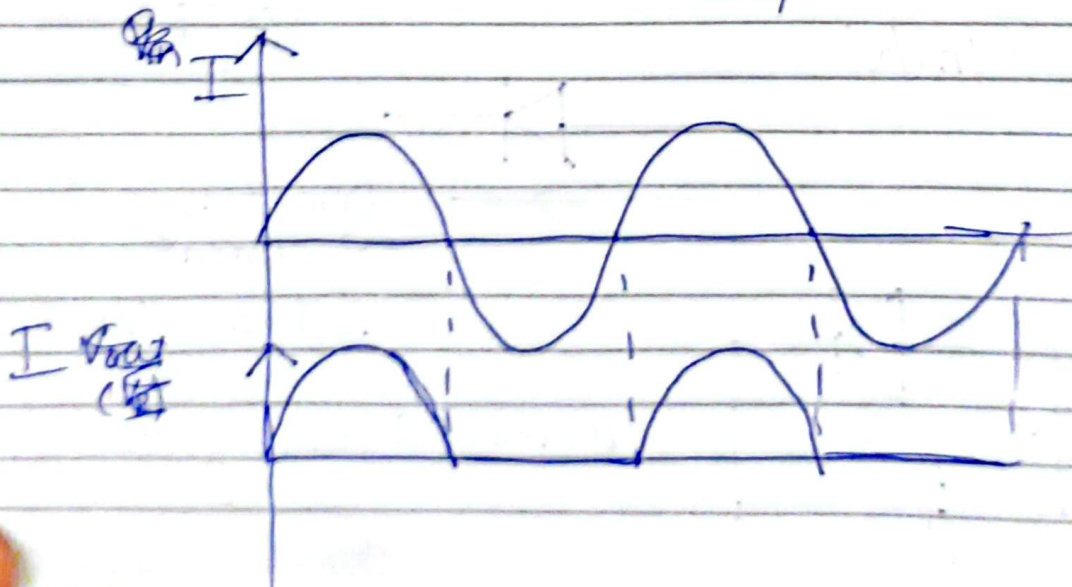
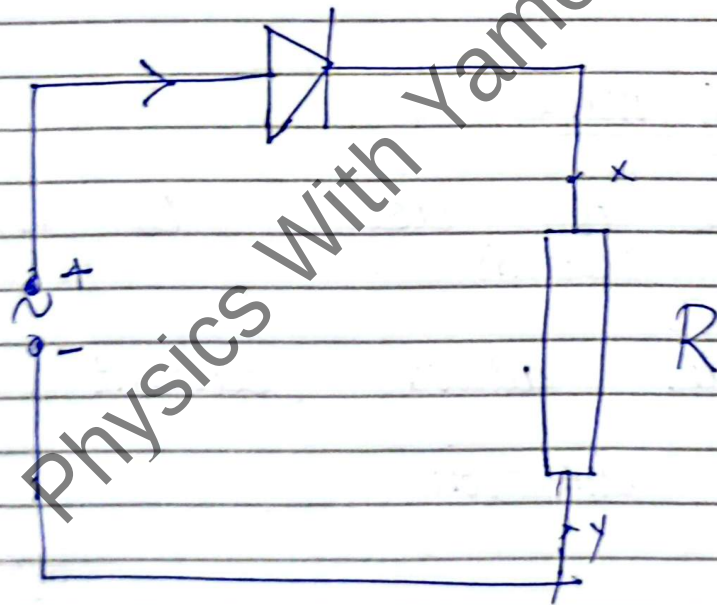
Direction of current that is allowed

current will flow



No current
will flow

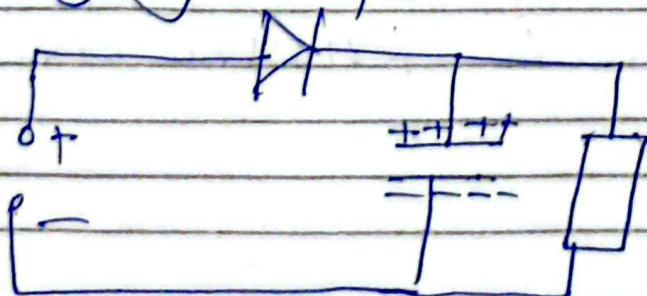
Circuit of half Wave Rectification



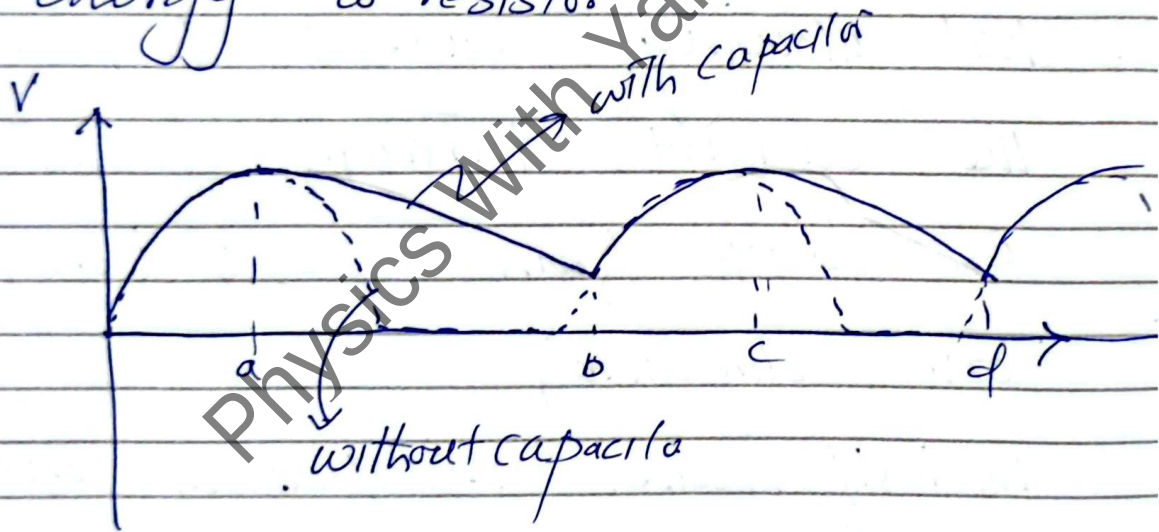
During positive half cycle of input current will flow through the diode and through resistor from x to y.
During negative half cycle of input no current will flow through the resistor as diode is reverse biased, not allowing current to pass. The current through resistor is always from $x \rightarrow y$ (only one direction).

In half wave rectification one cycle is stopped.

Applying Capacitor parallel to R



During positive half cycle capacitor will charge when during negative half cycle there is no current through the diode so capacitor will discharge through R . Now it will supply energy to resistor.

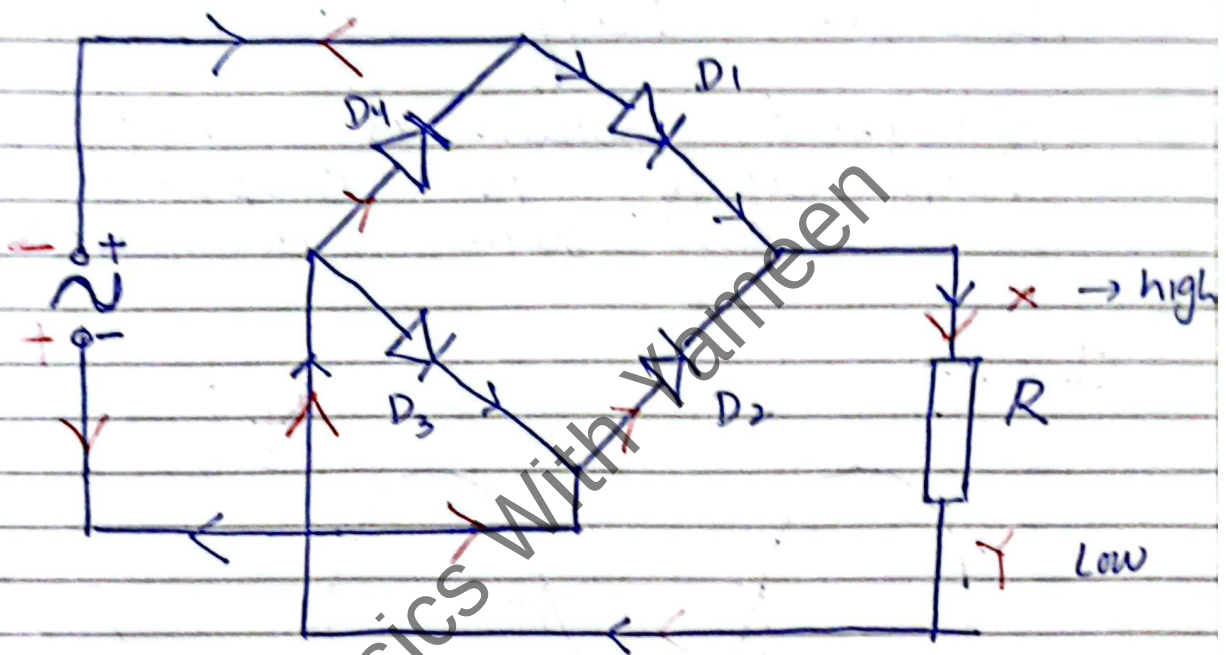


from a to b capacitor discharges
c " d

This process is known as
smoothing

Full wave rectification

Four diodes are used

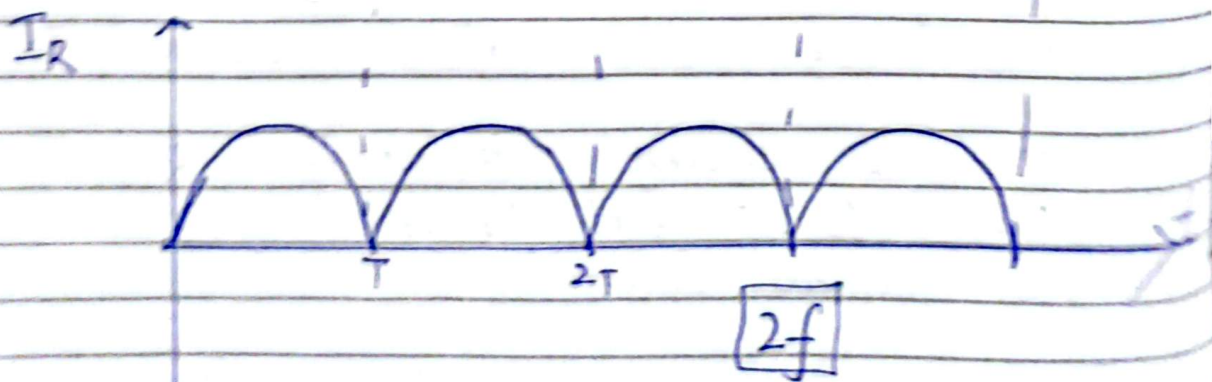
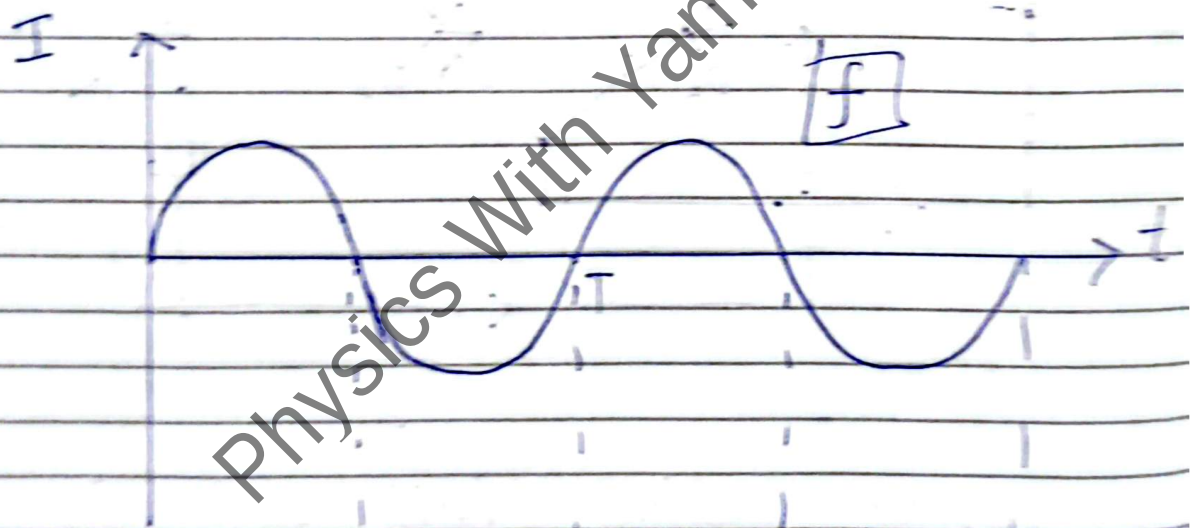


During positive half cycle D_1 and D_3 will allow current to pass so current through R is from x to y but D_2 and D_4 will not allow current.

During negative half cycle D_2 and D_4 will forward biased (allow current) but D_1 and D_3 will not. So direction

of current in this cycle
is also from x to y in
Resistor R .

In full wave rectification
negative half cycle is
reversed whereas negative
half cycle is stopped
in half wave rectification.

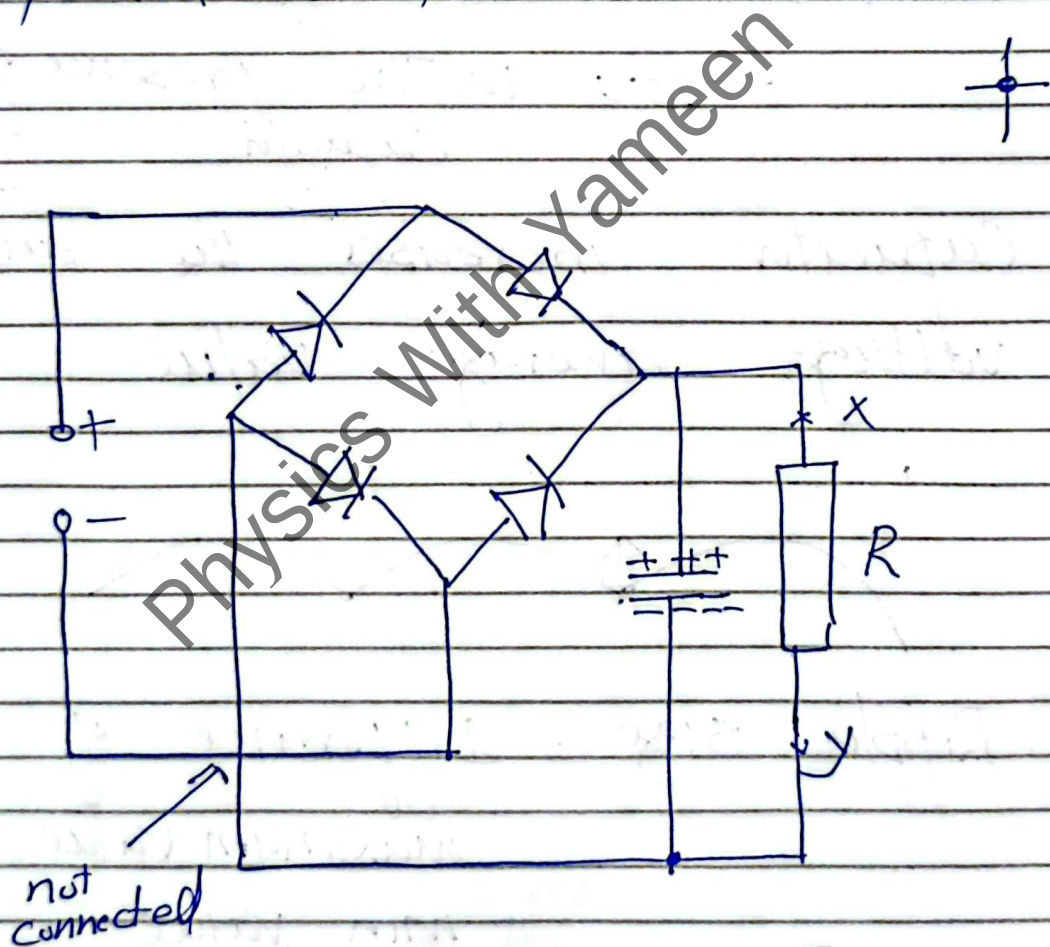


Frequency of rectified (Full wave) is
two times the frequency of AC

Smoothing

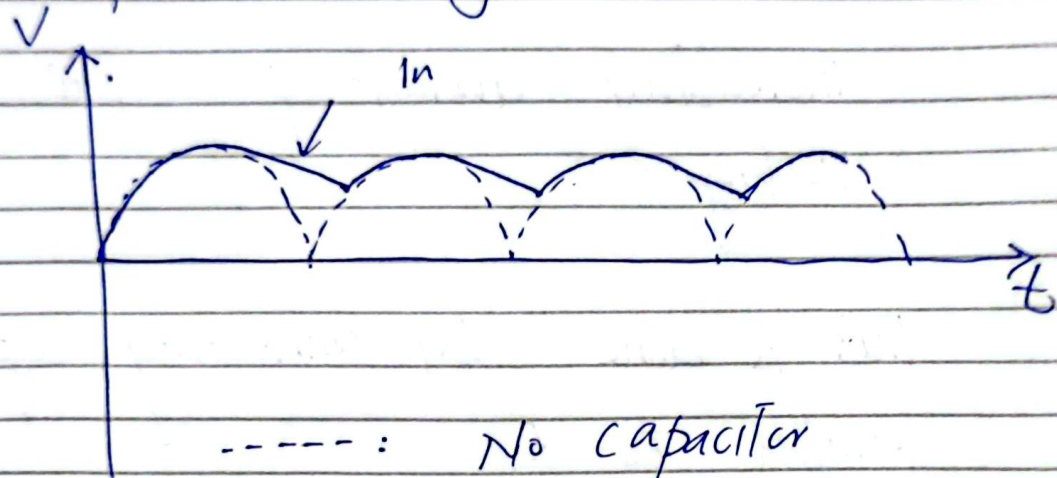
Increasing average of output voltage

This is done by connecting capacitor parallel to R



Capacitor will charge and then discharge through R

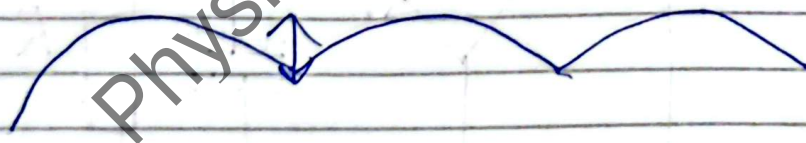
output voltage



----- : No capacitor

—— : In The presence of capacitor

Capacitor increases the output voltage average value



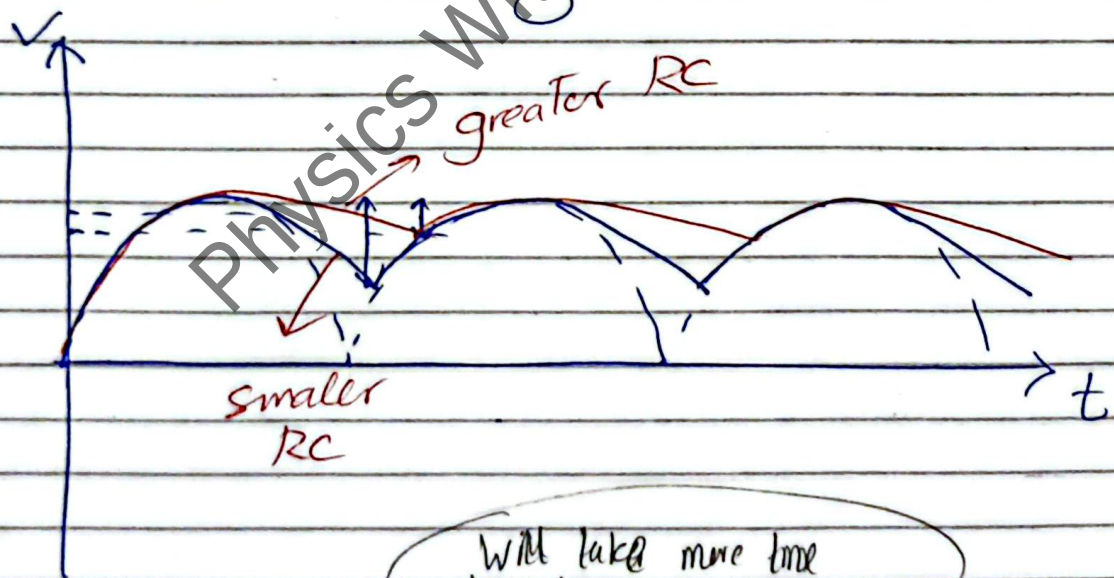
Ripple size : Difference of maximum value and min value

Capacitor decreases ripple size of output voltage

Effect of value of R and C on ripple size.

You know $\tau = RC$ is

time constant of this product is increased capacitor will discharge slowly so ripple size will decrease even it be zero



Will take more time to discharge

Capacitance

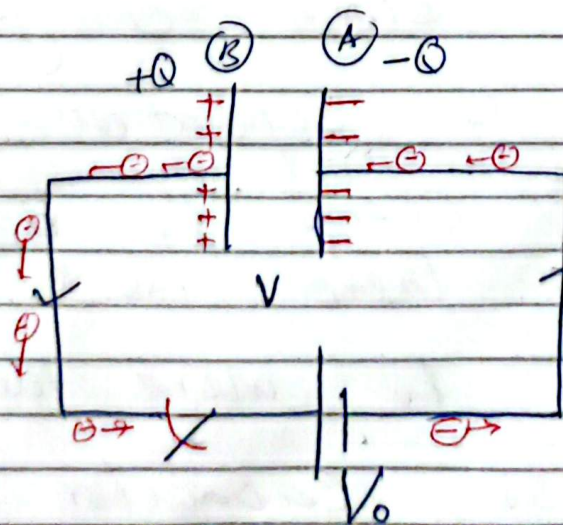
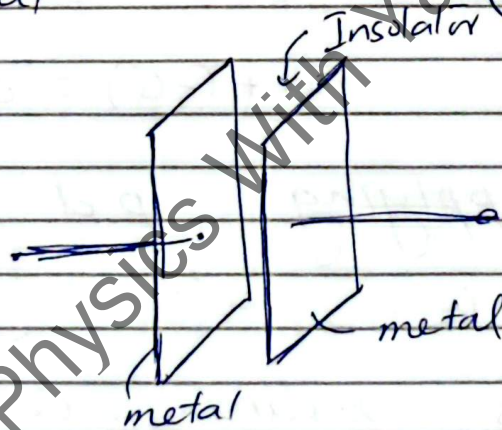
Capacitor:

A device/component that stores energy in the form of electric field.

Symbol



It is made up of two metal plates which are held parallel and they are separated by insulating material.



When switch is closed

electrons flow towards plate A. It becomes $\ominus$ charge

when switch is closed:

Electrons move away from plate B so it becomes $\oplus$ charge

Flow of electrons will stop when potential difference between plates becomes equal to emf of battery.

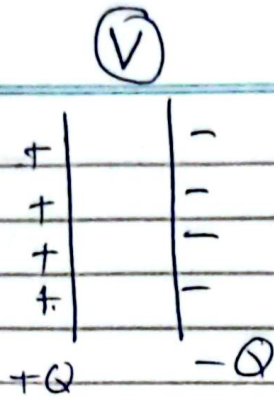
total charge on both plates =

$$Q + (-Q) = 0$$

Before applying p.d total charge was also zero

so this means capacitor does not store charge

But there is separation of $\oplus$ and $\ominus$ charge so this means there work has been done to move them apart so capacitor stores energy.



Due to charge on both plates
there is a potential difference
between them

$$Q \propto V$$

$$Q = CV$$

Where C is known as
capacitance

Capacitance

$$C = \frac{Q}{V}$$

Ratio of charge to potential
difference across capacitor.

unit is Farad (F)

Define Farad (F)

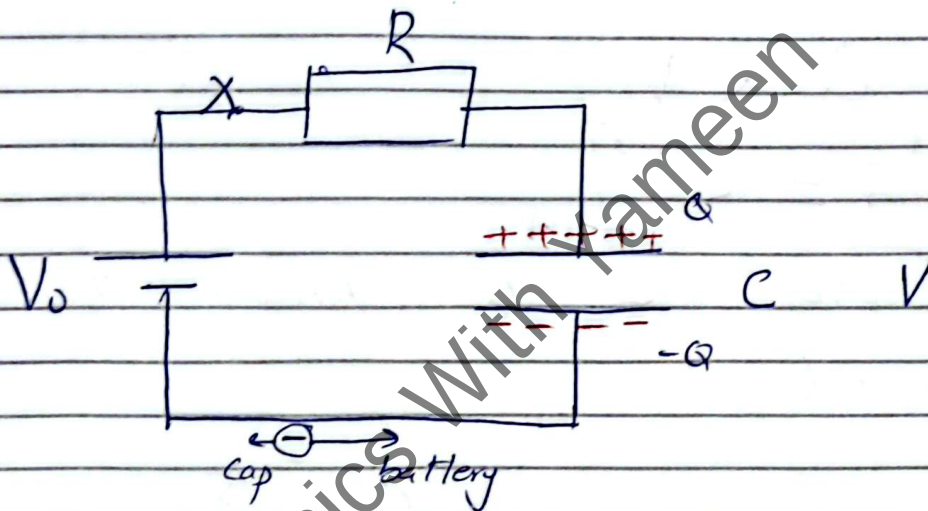
$$F = \frac{\text{Coulomb}}{\text{Volt}}$$

$$\boxed{C/V}$$

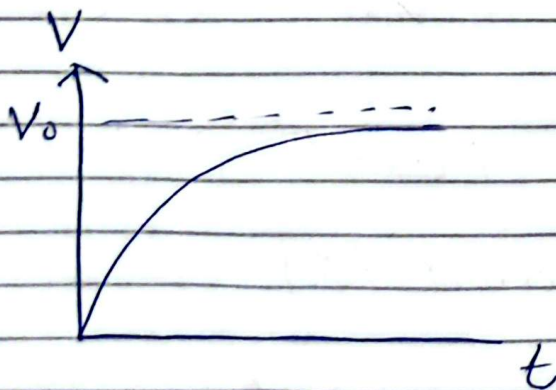
Micro farad $\rightarrow \mu F \rightarrow 10^{-6} F$

Nano farad $\rightarrow nF \rightarrow 10^{-9} F$

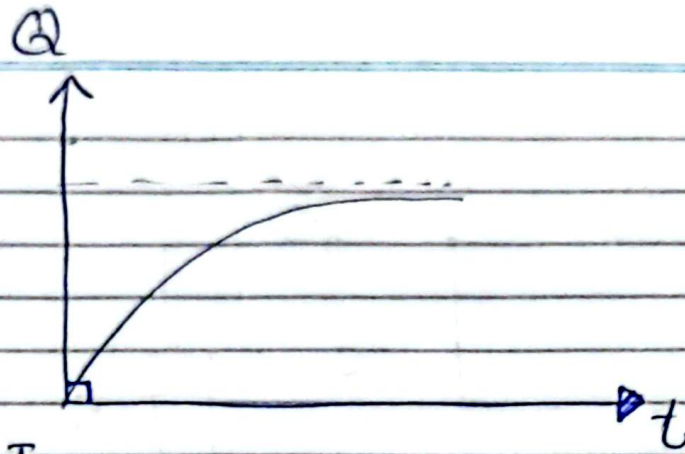
If we apply resistor in series to the circuit capacitor charging time will be controlled



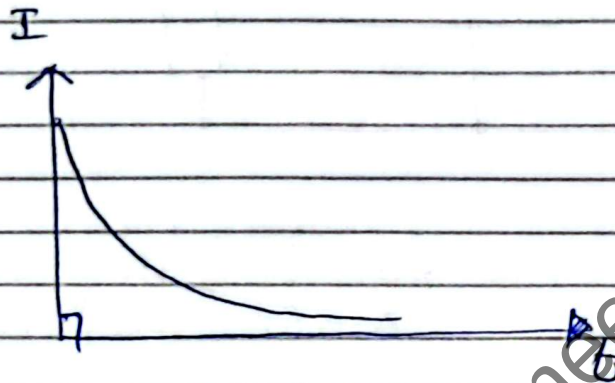
Voltage across capacitor will increase with time



V : Voltage across the capacitor

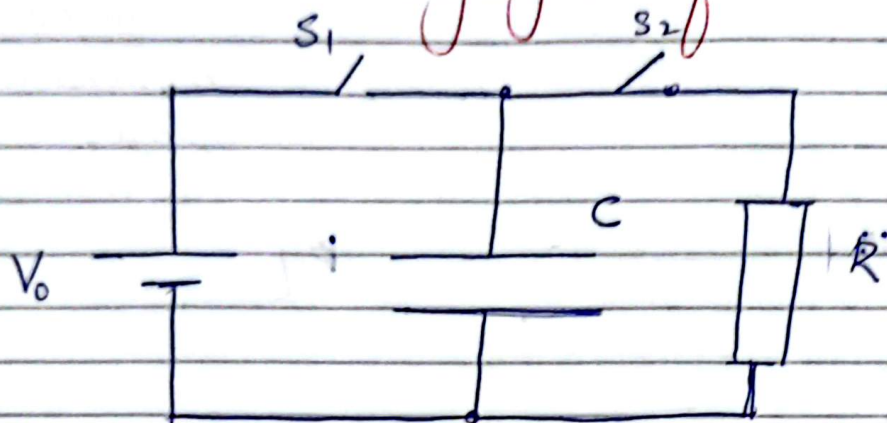


Rate of
Charge will
decrease with
time.



Physics With Yameen

Discharging of a capacitor



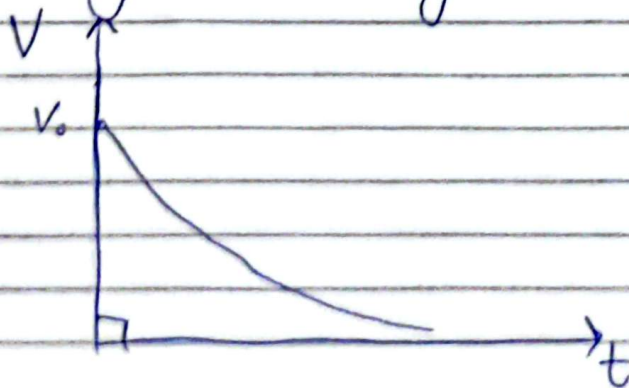
When S_1 is closed and S_2 is open the capacitor will charge upto V_0 .

When S_1 is open and S_2 is closed the capacitor will discharge through R .

The voltage across the capacitor will decrease as charge decreases.

$$V = V_0 e^{-\frac{t}{RC}}$$

This is voltage across capacitor during discharge at time t .



Rate of decrease of voltage is decreasing.

Time constant

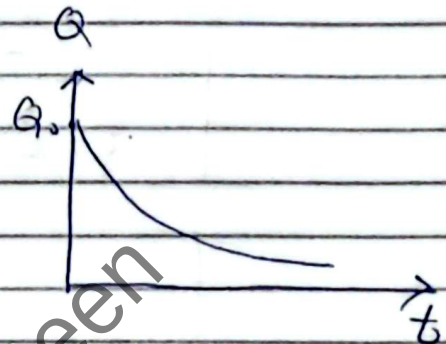
product of R and C

$$\therefore \tau = RC$$

unit: second

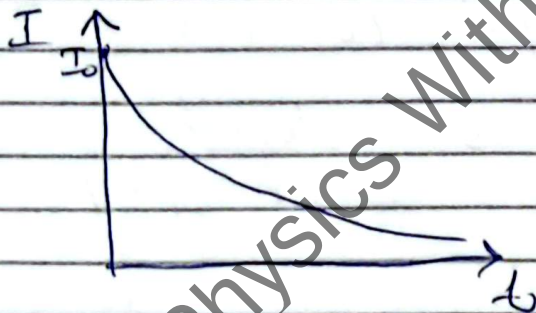
Equation of charge

$$Q = Q_0 e^{-\frac{t}{RC}}$$



Current

$$I = I_0 e^{-\frac{t}{RC}}$$

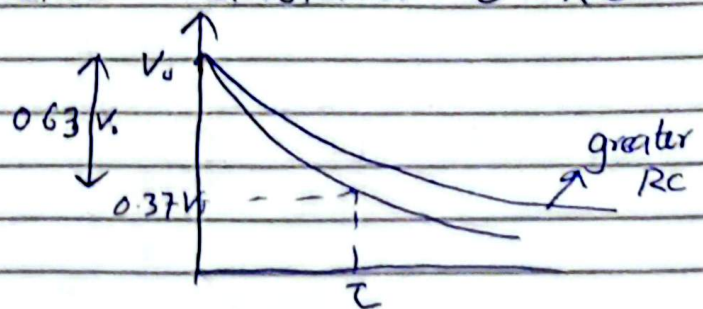


$$V = V_0 e^{-\frac{t}{RC}}$$

after one time constant $t = RC$

$$V = V_0 e^{-1}$$

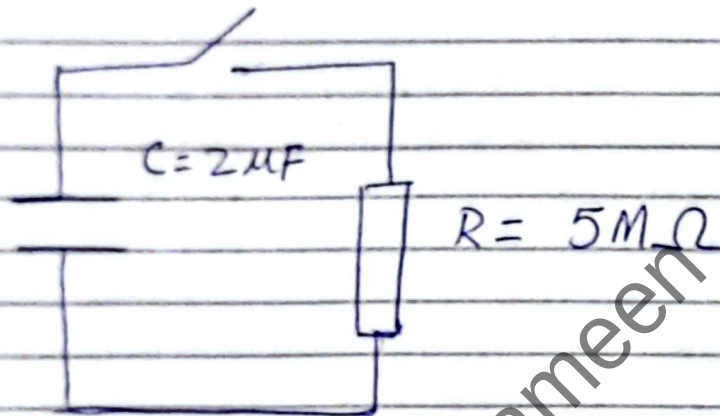
$$V = 0.37 V_0$$



In one time constant capacitor

discharges by 63%

Example



Capacitor is charged to 10 V

When switch is closed
capacitor starts discharging

$$V = V_0 e^{-\frac{t}{RC}}$$

$$V_0 = 10 \text{ V}$$

$$V = 10 e^{-\frac{t}{5 \times 10^6 \times 2 \times 10^{-6}}}$$

Convert
R into Ω
C " F

$$V = 10 e^{-\frac{t}{10}}$$

Find voltage at these times

t	V
0	10
1	9.0
2	8.2
3	7.4
⋮	
10	3.7
⋮	
50	0.1

$$V = 10 e^{-\frac{t}{10}}$$

$$V = 10 e^{-\frac{1}{10}}$$

Find time at which voltage across the capacitor is 6V

$$V = V_0 e^{-t/RC}$$

$$V = 10 e^{-\frac{t}{10}}$$

$$6 = 10 e^{-\frac{t}{10}}$$

$$\frac{6}{10} = e^{-\frac{t}{10}}$$

take $\ln$ on both sides

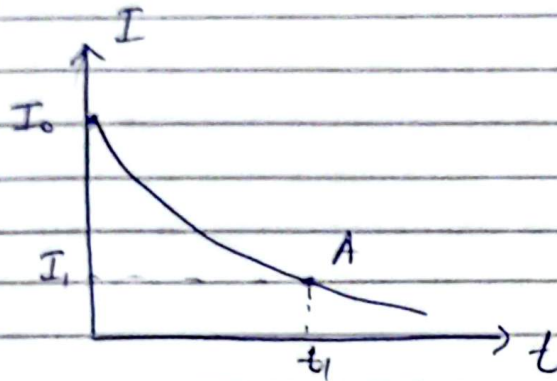
$$\ln\left(\frac{6}{10}\right) = \ln\left(e^{-\frac{t}{10}}\right)$$

$$\ln\left(\frac{6}{10}\right) = -\frac{t}{10} \ln e$$

$$t = 5.1 \text{ s}$$

Finding time constant from
graph : same for $(V-t)$ graph

$(\phi-t)$ "
 $(I-t)$ "



$$I = I_0 e^{-\frac{t}{\tau}}$$

$$RC = \tau$$

take any point
from the graph

lets take (A)

$$I_1 = I_0 e^{-\frac{t_1}{\tau}} \rightarrow \textcircled{1}$$

$$I_0 = \text{known}$$

$$I_1 = "$$

$$t_1 = "$$

Unknown τ

solve eq $\textcircled{1}$ to get τ

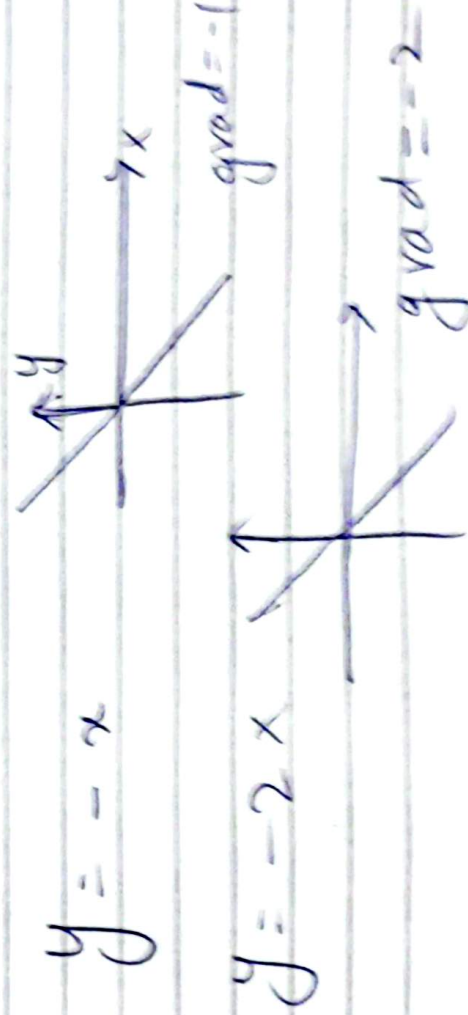
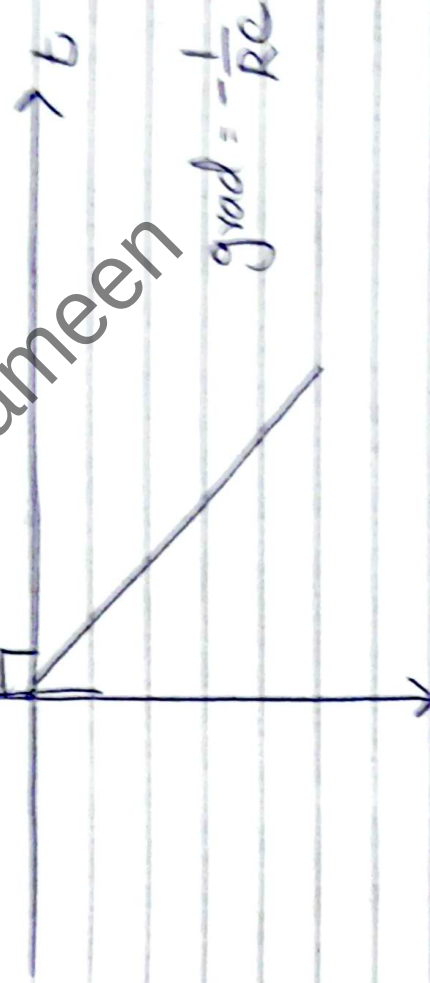
$$V = V_0 e^{-\frac{t}{RC}}$$

$$\frac{V}{V_0} = e^{-\frac{t}{RC}}$$

$$\ln\left(\frac{V}{V_0}\right) = \ln\left(e^{-\frac{t}{RC}}\right)$$

$$\ln\left(\frac{V}{V_0}\right) = -\frac{t}{RC} \ln e$$

Physics With Yameen

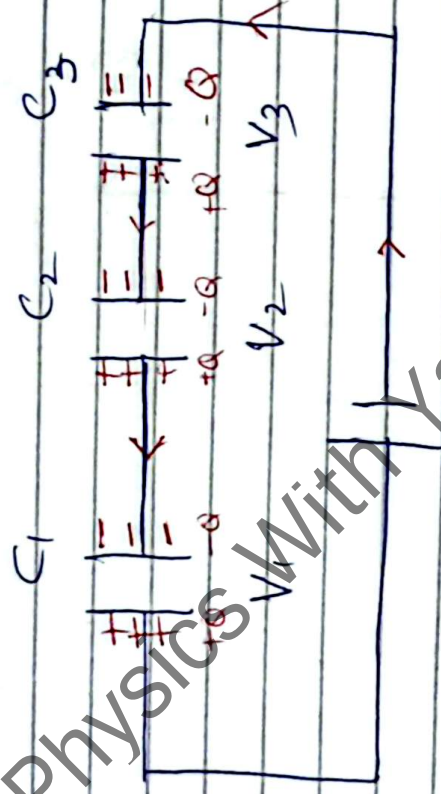


Combinations of capacitors

① Series

② parallel

Series



properties

* charge across each capacitor is same

* Voltage across each is different ($Q = CV$)

$$V = \frac{Q}{C}$$

Q is same $V \propto \frac{1}{C}$

as capacitors are different
so V is different

$$V = V_1 + V_2 + V_3$$

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

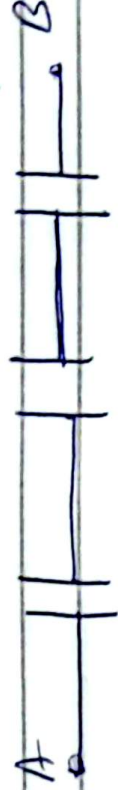
$$\boxed{\frac{1}{C_{tot}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

For capacitors in

Series

$$C_{tot} = \frac{\text{Product}}{\text{Sum}} = \frac{C_1 C_2}{C_1 + C_2}$$

20 μ F 40 μ F 60 μ F

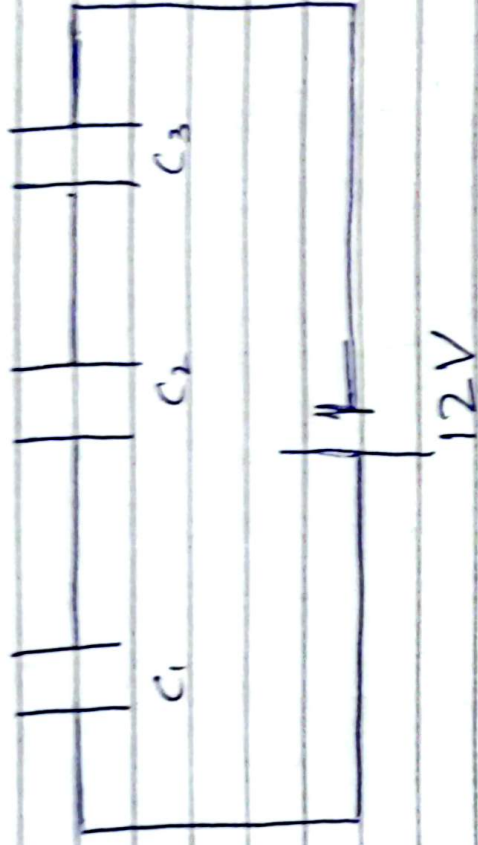


$$\frac{1}{C_{tot}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_{tot}} = \frac{1}{20} + \frac{1}{40} + \frac{1}{60}$$

$$C_{tot} = 10.9 \mu\text{F}$$

$6\mu F$ $4\mu F$ $8\mu F$



i) Find total capacitance

ii) Find total charge transferred by the battery

iii) Find charge across each capacitor

iv) Find potential difference across potential difference.

$$\frac{1}{C_{tot}} = \frac{1}{6} + \frac{1}{4} + \frac{1}{8}$$

$$C_{tot} = 1.85 \mu F \quad \text{or} \quad 1.85 \times 10^{-6} F$$

$$Q_{tot} = C_{tot} V$$

$$Q_{tot} = 1.85 \times 12 = 22.2 \mu C$$

Charge across each cap is
same as total charge.

$$Q = 22.2 \mu\text{C}$$

Voltage across each

$$V_1 = \frac{Q}{C} = \frac{22.2 \mu\text{C}}{6 \mu\text{F}}$$

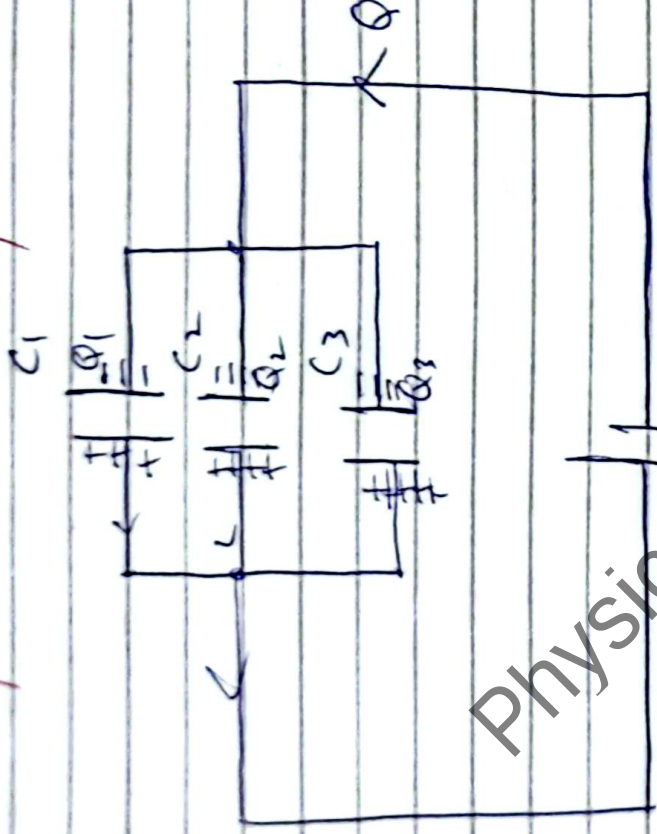
$$V_1 = \frac{22.2 \times 10^{-6} \text{C}}{6 \times 10^{-6} \text{F}}$$

$$V_1 = 3.7 \text{ V}$$

$$V_2 = \frac{Q}{C_2} = \frac{22.2}{4}$$

$$V_3 = \frac{Q}{C_3} = \frac{22.2}{8}$$

Capacitors in parallel.



Properties

- * p.d across each is same (V)
- * $Q = CV$ charge across each is different as V is same but C is different

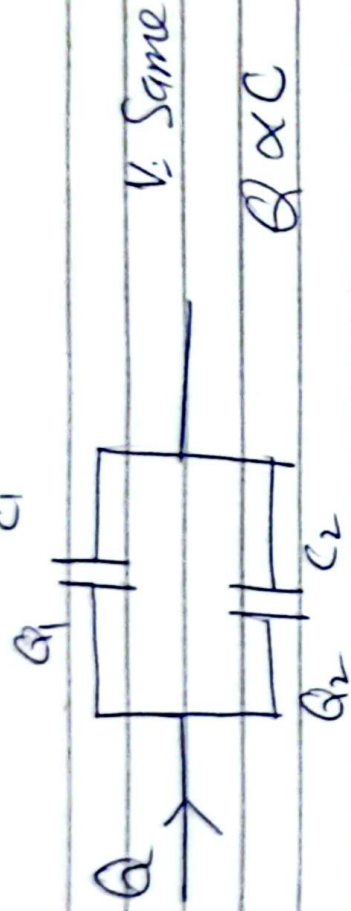
$$Q \propto C$$

$$Q = Q_1 + Q_2 + Q_3$$

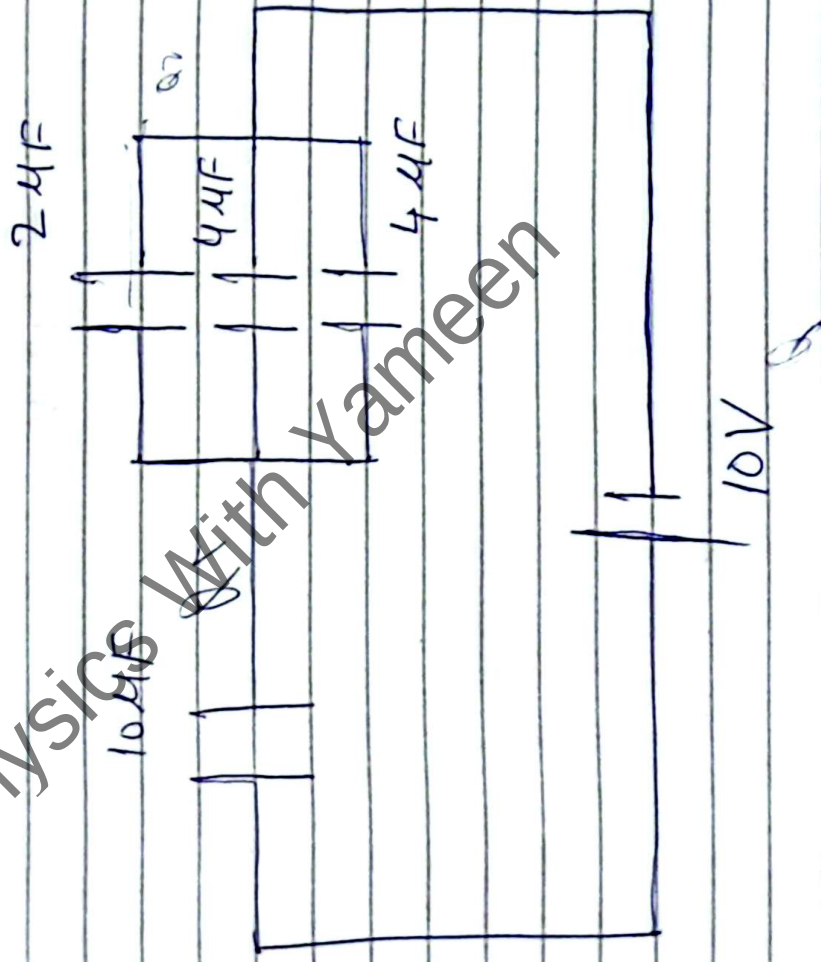
$$Q = C_1 V + C_2 V + C_3 V$$

$$\frac{Q}{V} = C_1 + C_2 + C_3$$
$$\boxed{C_{\text{tot}} = C_1 + C_2 + C_3}$$

Charge dividing



$$Q_1 = \frac{C_1}{C_1 + C_2} \times Q \quad Q = \text{total}$$



Find

- i) total capacitance
- ii) total charge
- iii) charge across each cap

iv) Voltage across each capacitor

2, 4, 4, are in parallel



$$\frac{1}{C_{tot}} = \frac{1}{10} + \frac{1}{10}$$

$$\frac{1}{C_{tot}} = \frac{2}{10}$$

$$C_{tot} = 5\mu F$$

$$Q = C_{tot} \times V$$

$$= 5 \times 10$$

$$Q = 50\mu C$$

cap 10 μF will have 50 μC charge -

but charge across 2, 4, 4 will distribute

$$V_{10\mu F} = \frac{Q}{C}$$

$$= \frac{50\mu C}{10\mu F} = 5V$$

$$Q_{2\mu F} = 5 \times 2 = 10\mu C$$

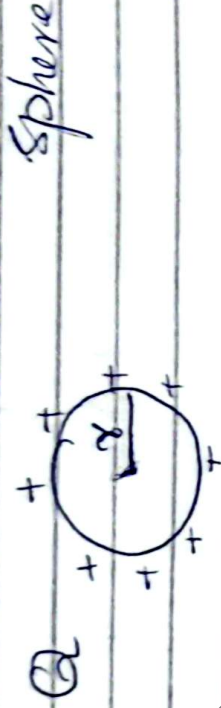
$$Q_{4\mu F} = 4 \times 5 = 20\mu C$$

$$Q_{4\mu F} = 4 \times 5 = 20\mu C$$

Charged Spherical conductors

as capacitor.

Example: positively charged



We assume the charge Q is at the center

Electric potential at surface

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

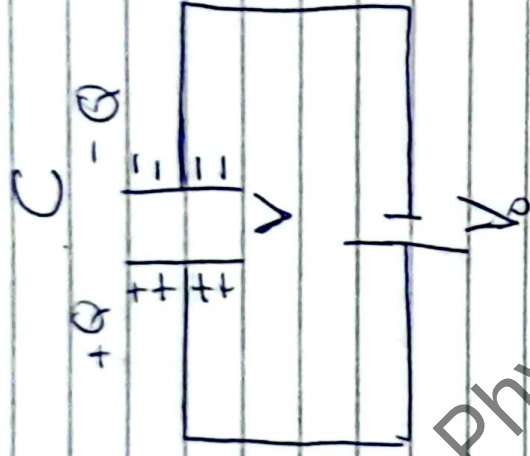
$$Q = CV$$

$$\frac{Q}{V} = C$$

$$4\pi\epsilon_0 r = \frac{Q}{V}$$

$$4\pi\epsilon_0 r = C$$

Energy stored in a capacitor.



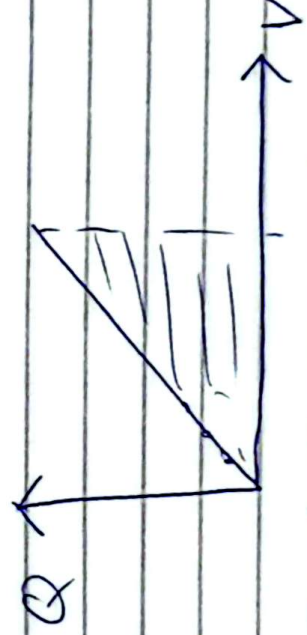
Charge on each plate is equal in magnitude but opposite in sign

battery does work to separate $\oplus$ and $\ominus$ charges. This work done is stored in the form of electric potential energy.

$$Q = CV$$

$$V = \frac{W}{Q}$$

$$W = VQ$$



Area is equal to work done = $\frac{1}{2} QV$

$$\text{Energy stored} = \frac{1}{2} QV \rightarrow (1)$$

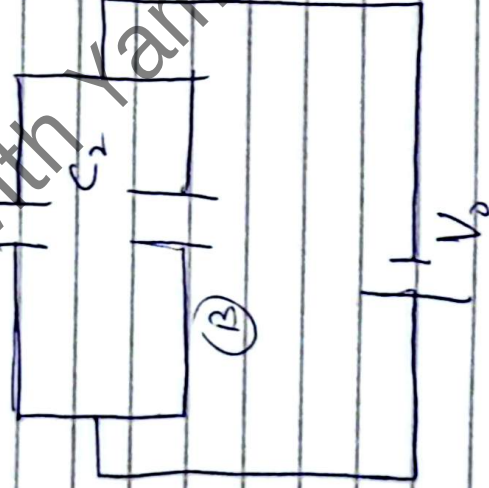
We know $Q = CV$

put in (1)

$$\text{Energy} = \frac{1}{2} CV \cdot V$$

$$\boxed{\text{Energy} = \frac{1}{2} CV^2}$$

(A)

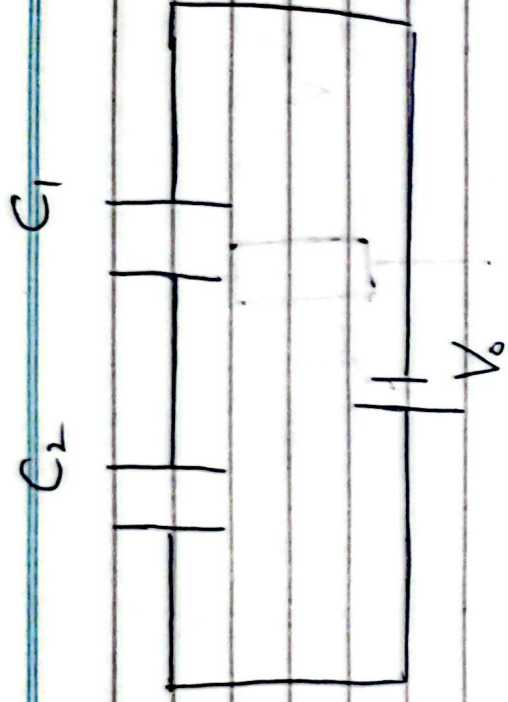


$E \propto C$
in this case
as V is same

Energy stored in A : $\frac{1}{2} C_1 V_0^2$

" " B : $\frac{1}{2} C_2 V_0^2$

$$\text{Ratio} \left[\frac{E_A}{E_B} = \frac{C_1}{C_2} \right]$$



In series charge is same.

but Voltage is different

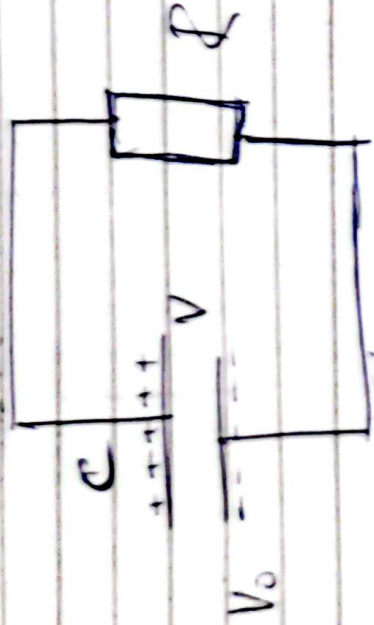
$$V \propto \frac{1}{C}$$

$\therefore$

$$E = \frac{1}{2} QV \quad Q: \text{same}$$

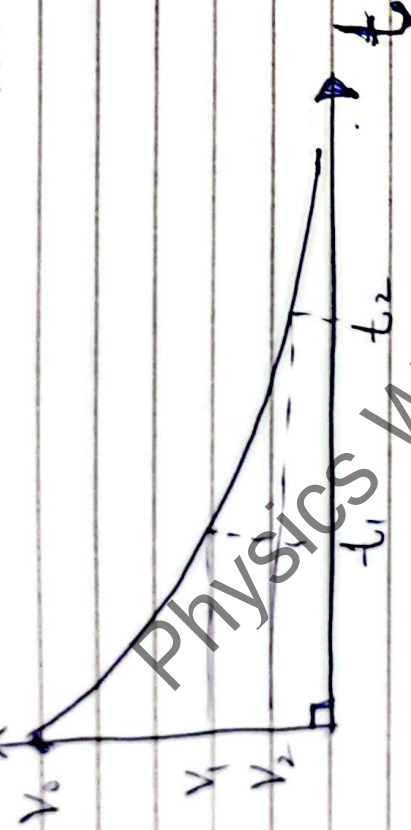
$$E \propto \frac{1}{C}$$

$$\boxed{\frac{E_{C1}}{E_{C2}} = \frac{C_2}{C_1}}$$



C : discharging

V_0 : initial V_0



Find Loss of Energy from

t_1 to t_2 ?

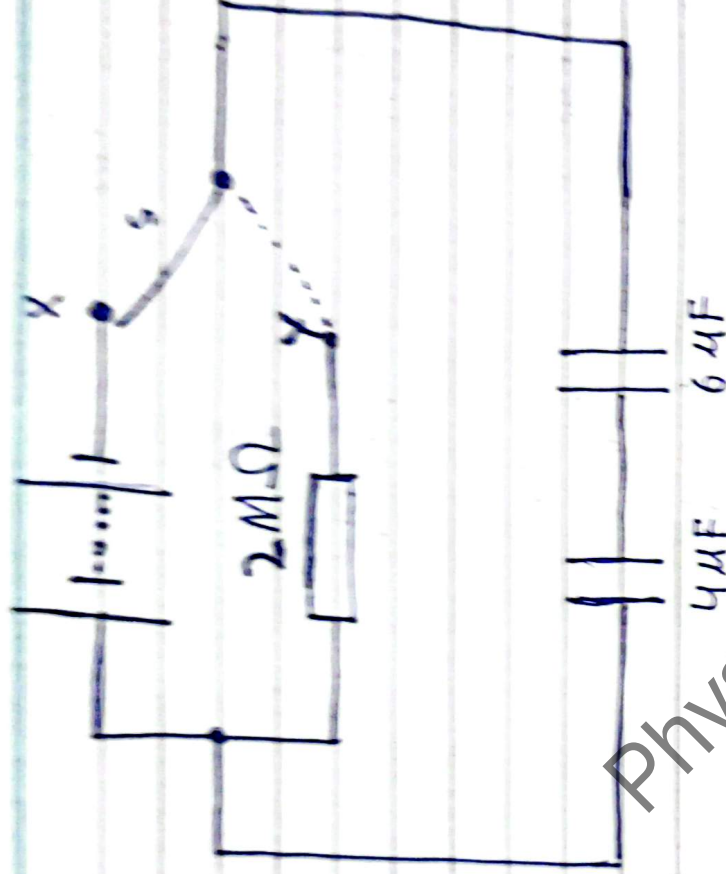
at t_1 : $E_1 = \frac{1}{2} C V_1^2$

at t_2 : $E_2 = \frac{1}{2} C V_2^2$

Loss of energy = $\frac{1}{2} C V_1^2 - \frac{1}{2} C V_2^2$

" " = $\frac{1}{2} C (V_1^2 - V_2^2)$

12V



When switch is at X both capacitor will charge

i) find voltage across each capacitor.

$$C_{tot} = \left(\frac{1}{C_1} + \frac{1}{C_2} \right)^{-1}$$

$$\frac{1}{C_{tot}} = \frac{1}{4} + \frac{1}{6}$$

$$\frac{1}{C_{tot}} = \frac{3}{12} + \frac{2}{12}$$

$$C_{tot} = \frac{12}{5} = 2.4 \mu F$$

$$Q_{tot} = C_{tot} \times V = 2.4 \times 12 = 28.8 \mu C$$

$$V_{4F} = \frac{Q}{C}$$

$$= \frac{28.8}{4}$$

$$V_{4F} = 7.2 \text{ V}$$

$$V_{6\mu F} = \frac{28.8}{6}$$

$$V_{6\mu F} = 4.8 \text{ V}$$

When switch is connected
is at y then capacitors
will discharge.

Voltage across both cap
will decrease

$$V = V_0 e^{-\frac{t}{RC_{tot}}}$$

$$V = 12 e^{-\frac{t}{2 \times 10^{-6} \times 2.4 \times 10^{-6}}}$$

$$V_{tot} = 12 \cdot e^{-\frac{t}{4.8}}$$

find total voltage at

$$t = 1s, \quad t = 10s, \quad t = 15s$$

$$V = 12 e^{-\frac{1}{4.8}} \quad V = 12 e^{-\frac{10}{4.8}}$$

$$V = 9.74V \quad V = 1.49V$$

at $t = 15s$:

$$V = 12 e^{-\frac{15}{4.8}}$$

$$V = 0.527V$$

if we have to find

voltage across any capacitor

find voltage across 4μF at

$$t = 2s$$

there are two methods

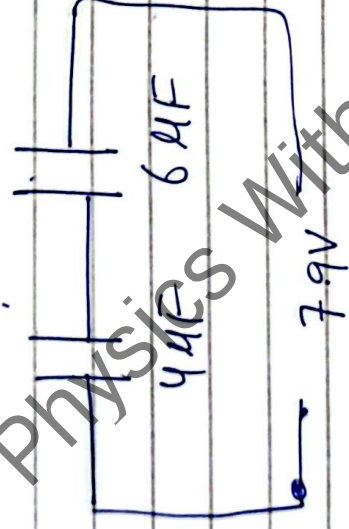
① Find total voltage after 2s

$$V = V_0 e^{-t/R_{\text{tot}} C_{\text{tot}}}$$

$$V = 12 e^{-\frac{2}{4.8}}$$

$$V = 7.9 \text{ V}$$

$V_1?$



$$V_1 = \frac{C_2 \times V_0}{C_1 + C_2}$$

$$V_1 = \frac{6}{6+4} \times 7.9$$

$$V_1 = 4.74 \text{ V}$$

2nd Method

Find initial voltage across

$$4 \mu\text{F} : V_{4\mu\text{F}} = 7.2 \text{ V}$$

Use This

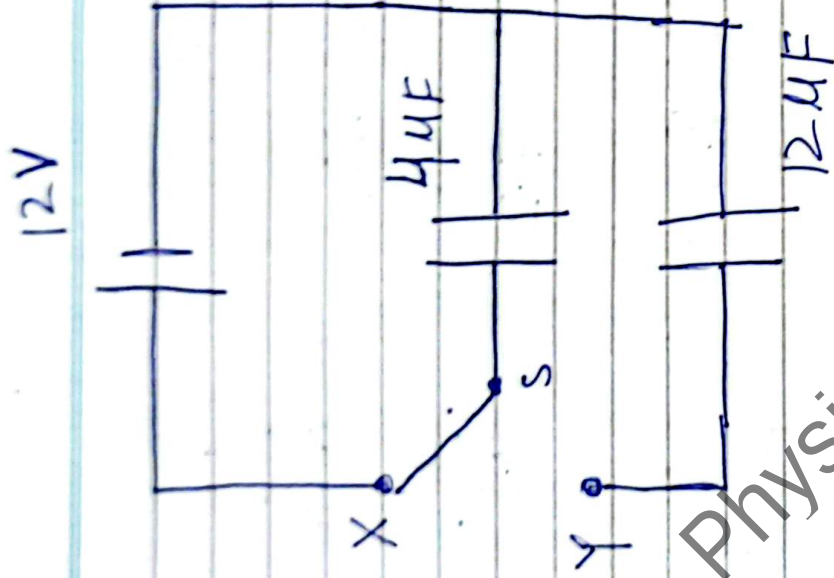
$$V_{4uF} = V_{0uF} e^{-\frac{t}{RC_{tot}}}$$

$$V_{4uF} = 7.2 e^{-\frac{2}{4.8}}$$

$$V_{4uF} = 4.74$$

Same as we got by 2nd method

Physics With Yameen



When switch is at X

4μF will get charge

$$Q = CV$$

$$Q = 4 \times 12 = 48 \mu C$$

When switch is connected at

Y 4μF will discharge

and 12μF will charge

This process will stop when
Potential difference

Find Charge across each will be different.

across both will be same.

If we have to find the final potential difference?

$$Q = Q_{4\mu F} + Q_{12\mu F}$$

$$Q = C_{4\mu F} V + C_{12\mu F} V$$

$$48 \mu F = 4 \mu F V + 12 \mu F V$$

$$48 \mu F = 16 \mu F V$$

$$48 \times 10^{-6} = 16 \times 10^{-6} V$$

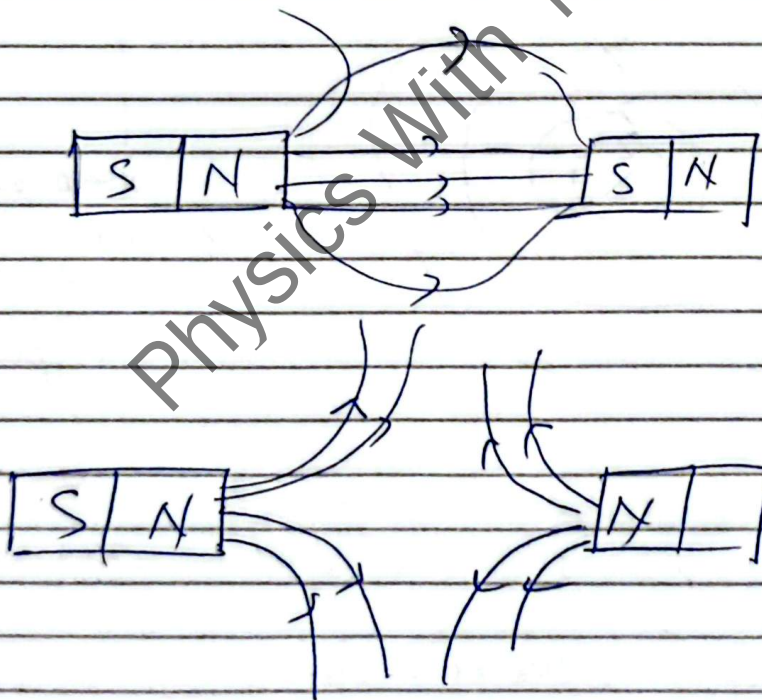
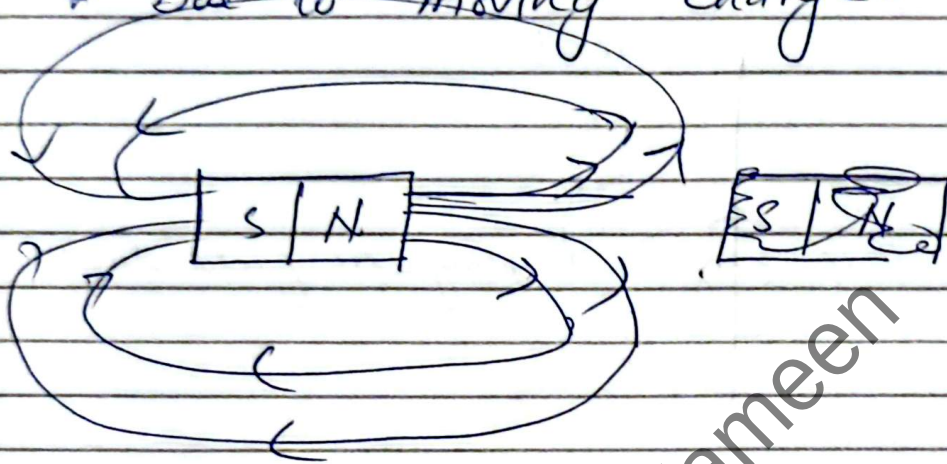
$$3 V = V$$

Magnetic fields

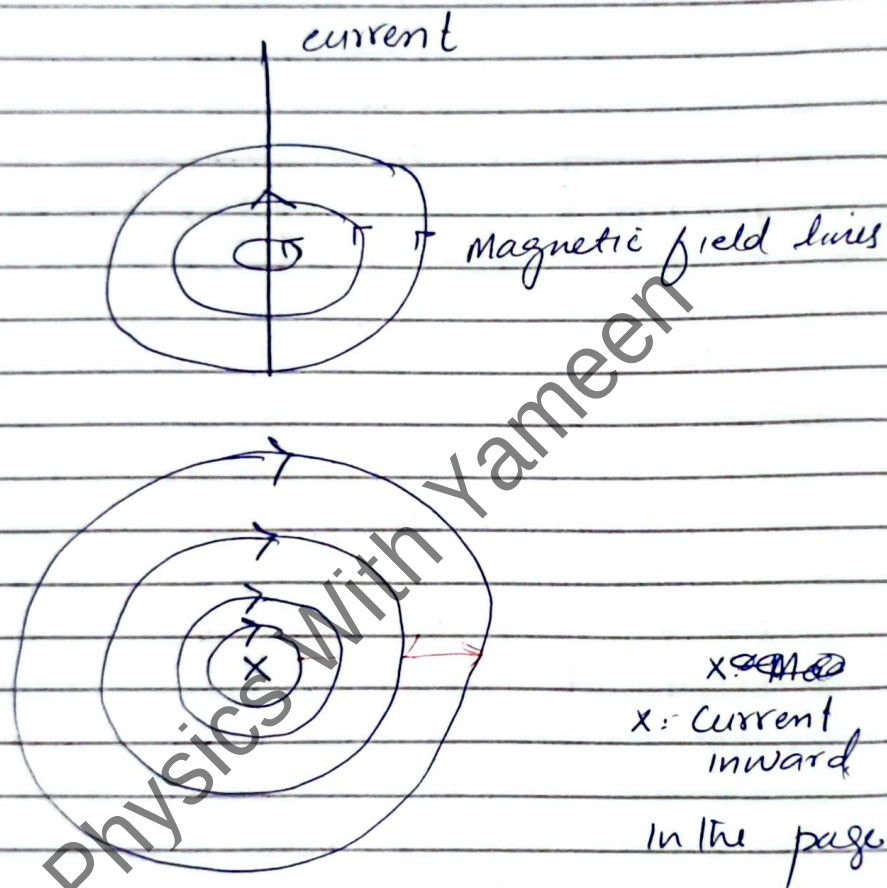
Magnetic field is due to

* permanent magnet

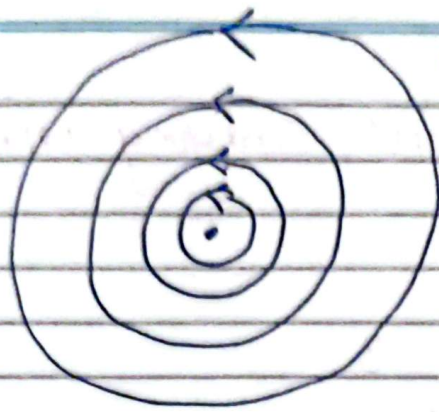
* Due to moving charges



Magnetic field due to current carrying wire

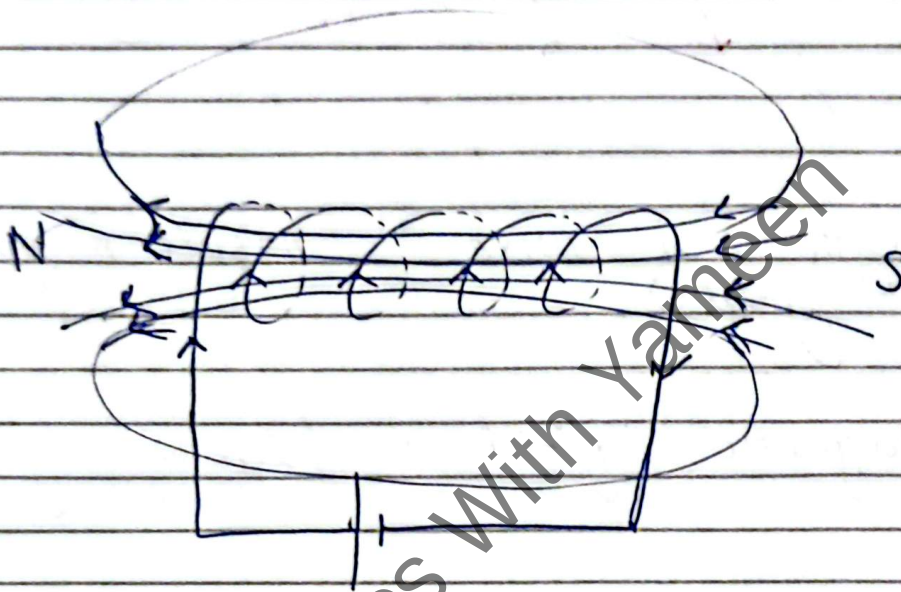


Distance between magnetic field lines should ~~decrease~~ ^{increase} due to weakening of magnetic field as you move away from the wire



Magnetic field.

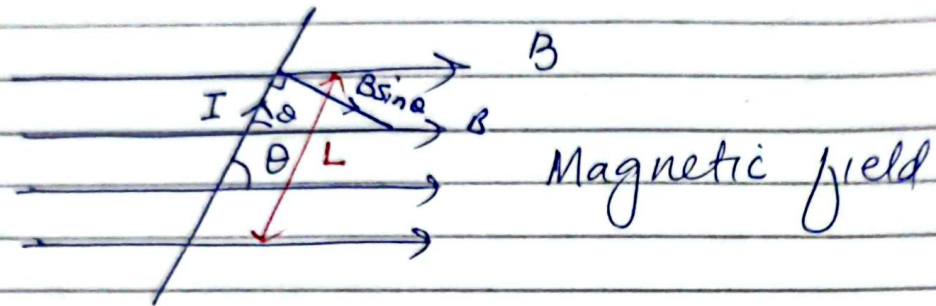
- current out of the page



Electromagnet

Grip the coil using your right hand fingers pointing in ~~current~~ current direction thumb will be pointing towards North side.

Force on current carrying wire



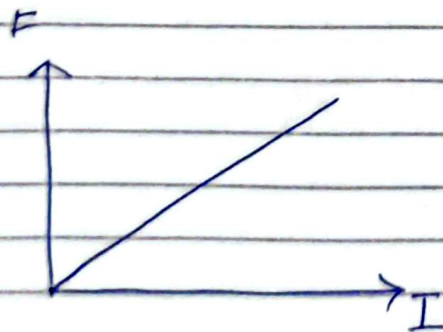
L = length of wire inside magnetic field

B = Magnetic field or flux density

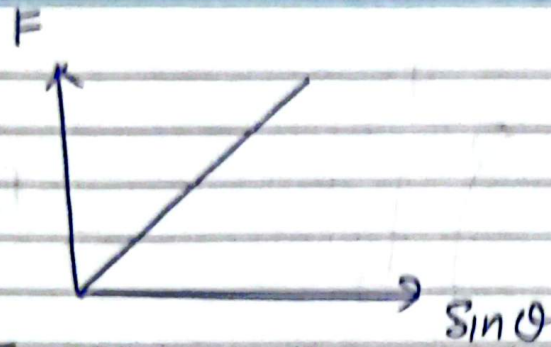
When a wire having current I is placed inside magnetic field it may experience force

θ : angle b/w wire and magnetic field

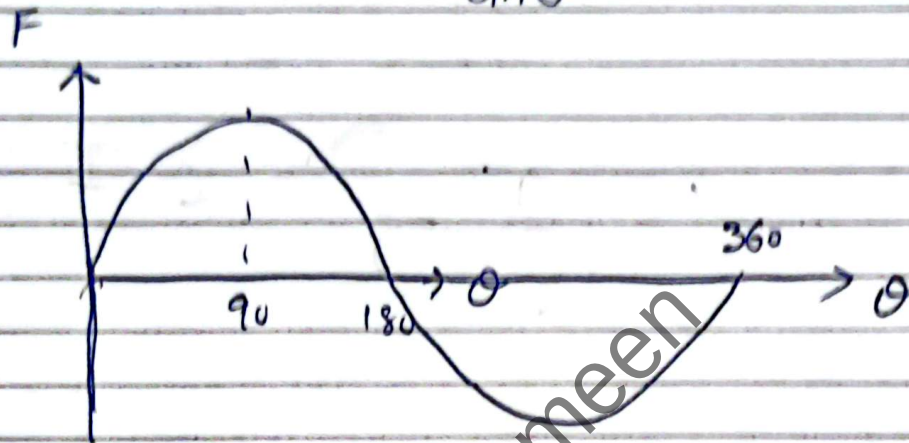
$$F = BIL \sin \theta$$



B, L, θ : constant

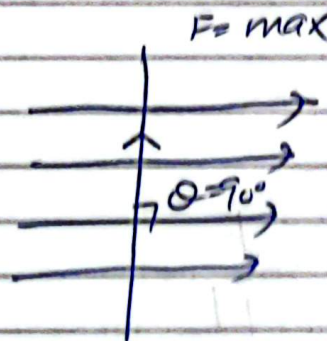


I, B, L constant



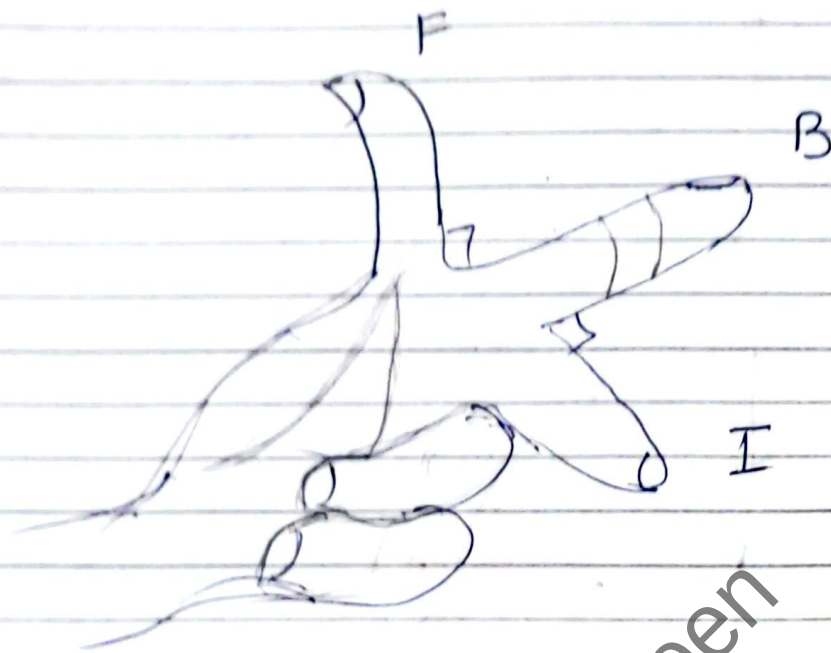
When $\theta = 90^\circ$ F is max

$$F_{\max} = BIL$$



Direction of force?

Fleming's left hand rule



$$F = BIL \sin \theta$$

$$B = \frac{F}{IL \sin \theta}$$

$$\theta = 90$$

$$B = \frac{F}{IL}$$

B: Magnetic flux density

Force per unit current
per unit length on

a wire placed perpendicular
to magnetic field

unit: Tesla (T)

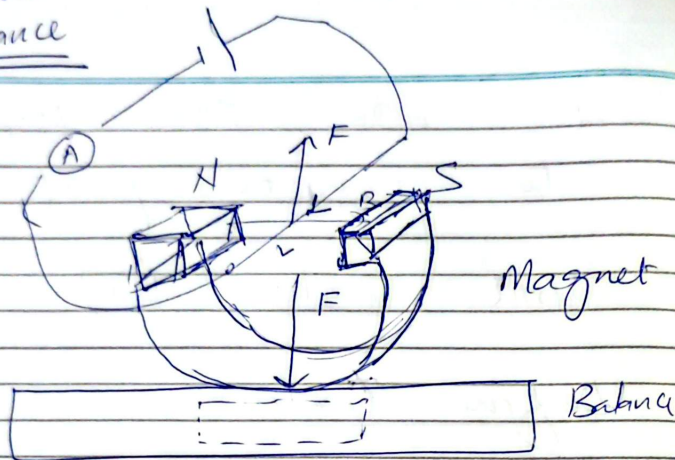
Define Tesla:

$$T = \text{N A}^{-1} \text{m}^{-1}$$

Newton per Ampere per meter

of a wire of length 1 meter
having current 1A placed
perpendicular to field experiences
1 Newton force. Then
magnetic flux density is
1 tesla.

Current Balance



According to Fleming's left hand rule
Force on wire
due to magnetic is acting
upward,

By Newton's Third law
Force on magnet will

be downward so balance
reading will increase

Δm : increase in balance
reading

$$\Delta mg = F$$

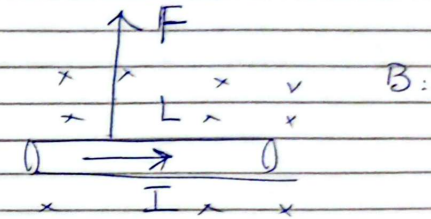
$$\theta = 90^\circ$$

$$F = BIL$$

$$\Delta mg = BIL$$

$$B = \frac{\Delta mg}{IL}$$

Force on moving ch
inside magnet



$$F = BIL \sin \theta$$

$$F = B \frac{Q}{t} L \sin \theta$$

$$F = B Q \frac{L}{t} \sin \theta$$

$$F = B Q v \sin \theta$$

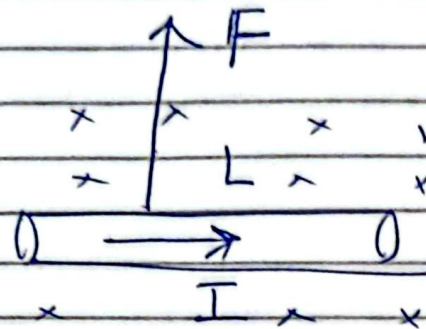
$$F = B n q v \sin \theta$$

if $n=1$

$$F = B q v \sin \theta$$

$$F = q v B \sin \theta$$

Force on moving charge
inside magnetised field.



B : Into the page.

$$F = BIL \sin \theta$$

$$F = B \frac{Q}{t} L \sin \theta$$

$$F = B Q \left[\frac{L}{t} \right] \sin \theta$$

$$F = B Q v \sin \theta$$

Q = total charge

$$F = B n q v \sin \theta$$

if $n=1$

$$F = B q v \sin \theta$$

$$F = q v B \sin \theta$$

If a charged particle has some component of velocity perpendicular to magnetic field then it will experience force

$$F = qvB \sin \theta$$

q : charge

v : speed or velocity

B : Magnetic flux density

θ : angle b/w velocity and magnetic field.

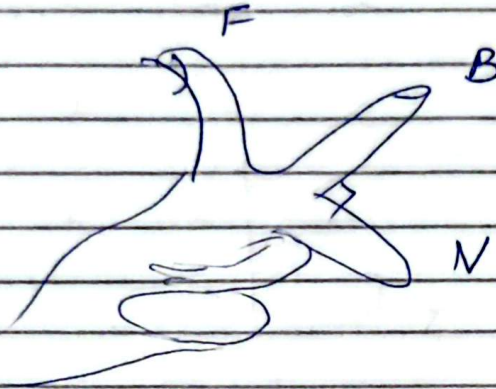
$$\theta = 0^\circ : F = 0$$

$$v = 0 : F = 0$$

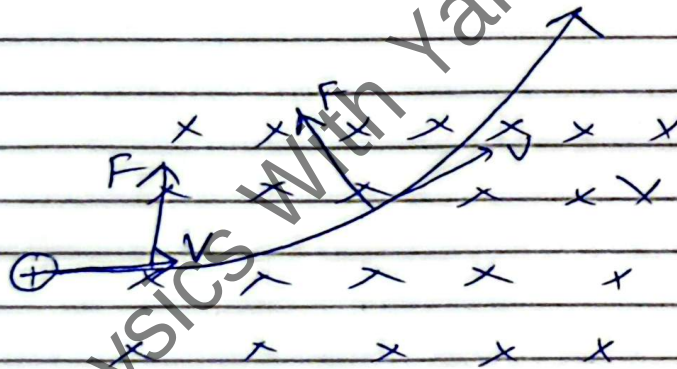
Direction of force?

If charge is (+ive)

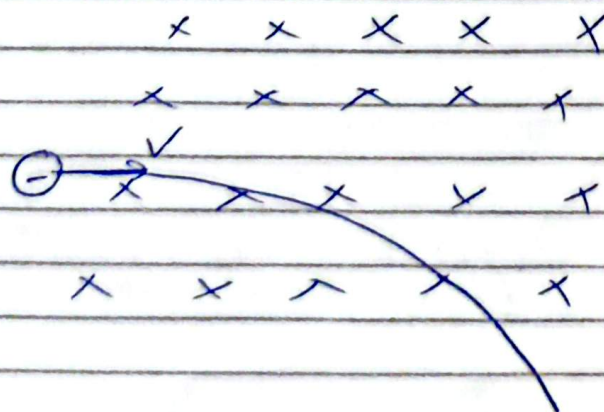
Use Fleming's left hand rule



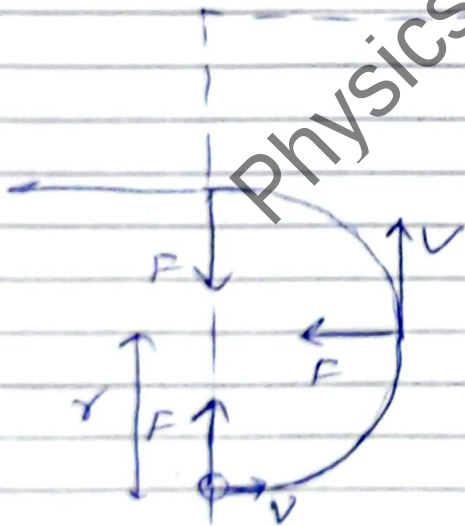
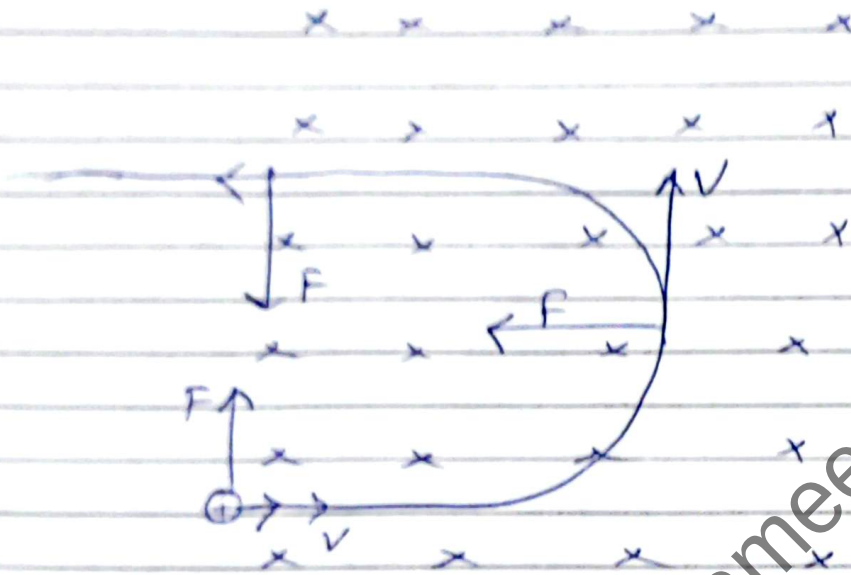
Q



B: Into the page



assume
Q to be
+ve and
use LHR
to find
F and
Then opposit
it



Magnetic field
into the page

Magnetic force is centripetal force.

$$F_M = F_c$$

$$qvB = \frac{mv^2}{r}$$

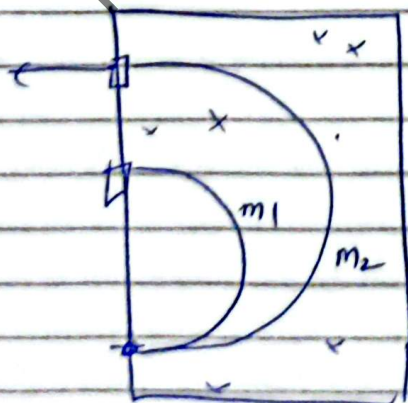
$$r = \frac{mv}{qvB}$$

$$r \propto m$$

$$r \propto v$$

$$r \propto \frac{1}{q}$$

$$r \propto \frac{1}{B}$$



Magnetic field
into the page

Charge same

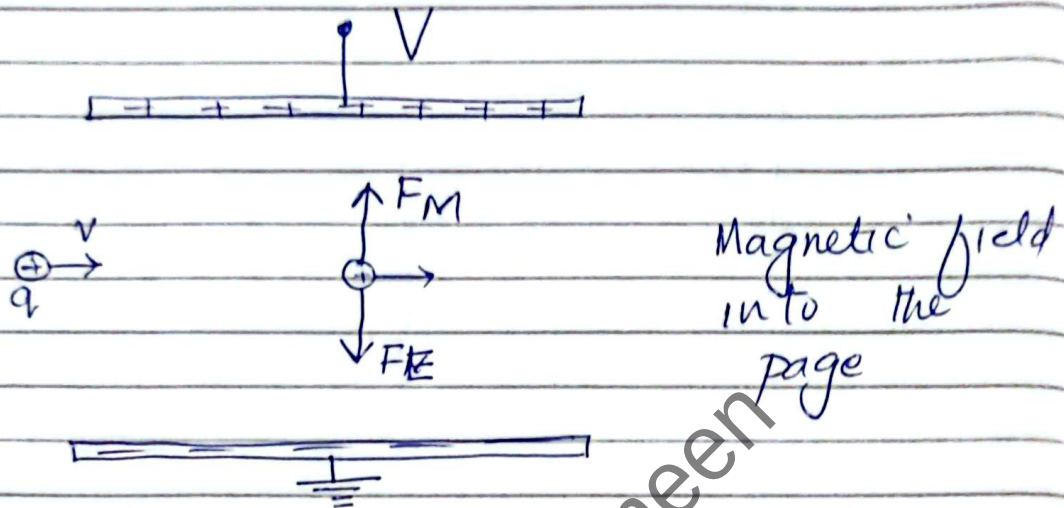
v ; Same

m , different

r diff

$$m_2 > m_1$$

Velocity selection



Magnetic field
into the
page

Magnetic field is into the
page

(+ive)
of charge particle q enters
in this region

Magnetic force is up
Electric field is applied
perpendicular to magnetic
field so electric force
is acting down

When

$$F_E = F_M$$

Charge particle will go
straight undeflected

$$F_E = Eq$$

$$F_M = qvB \sin \theta \quad \theta = 90^\circ$$

$$Eq = qvB$$

$$v = \frac{E}{B}$$

i) If $v > E/B$

$$F_M > F_E$$

Charge will

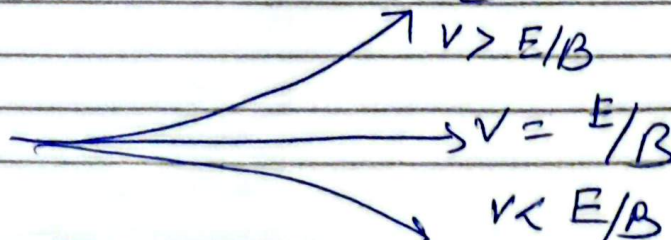
deflect upward

If $v < E/B$

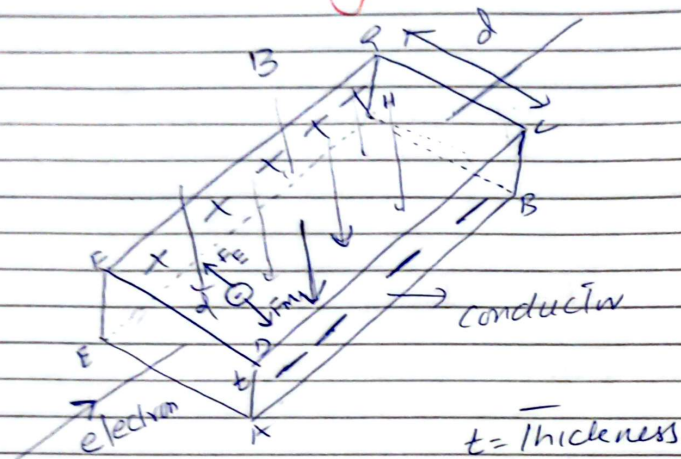
CRG

$$F_M < F_E$$

deflect down

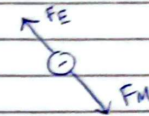


Hall Voltage



Magnetic field is applied perpendicular surface $F \perp B$
 moving electrons will experience magnetic force toward face ABCD
 so this face becomes negative and $E = \frac{V_H}{d}$ becomes positive
 Hence potential difference is established between these two faces known as hall voltage V_H

due to hall voltage is
 electron will also experience electric force



When these two forces equal then electron goes there will be no increase in hall voltage

$$F_M = F_E$$

$$qVB = Eq$$

$$VB = \frac{V_H}{d}$$

$$VB = \frac{V_H}{d}$$

$$VBd = V_H \rightarrow (1)$$

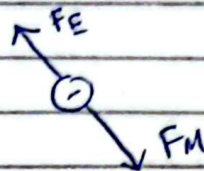
from As physics

$$A = db \quad \text{area of}$$

$$I = n d t a v$$

due
when hall voltage is 0

electron will also experience
electric force



When these two forces become
equal then electron goes straight
there will be no further
increase in hall voltage.

$$F_M = F_E$$

$$q v_B = E q$$

$$q v_B = \frac{V_H}{d} q$$

$$v_B = \frac{V_H}{d}$$

$$v_B d = V_H \rightarrow (1)$$

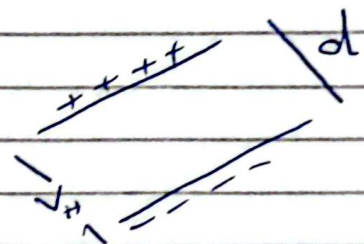
from AS physics

$$I = n A q v$$

$$A = d t$$

area of EFDA

$$I = n d t q v$$



$$V = \frac{I}{n d t a}$$

put in (1)

$$V_H = V B d$$

$$V_H = \frac{I B d}{n d t a}$$

$$V_H = \frac{B I}{n t a}$$

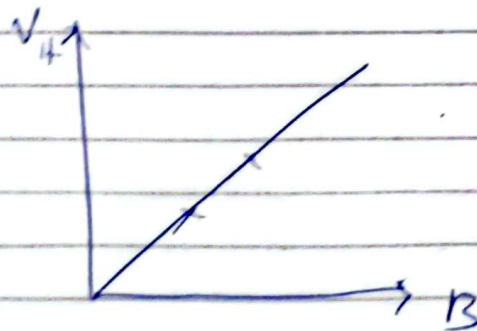
$$V_H \propto I$$

$$V_H \propto \frac{1}{n}$$

$$V_H \propto \frac{1}{t}$$

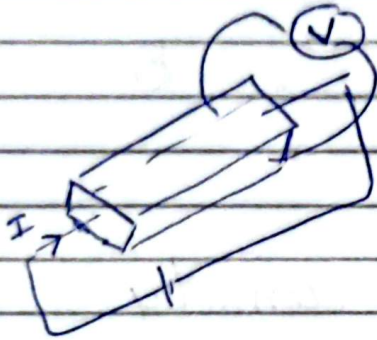
$$V_H \propto B$$

n : number of
= free electrons
per unit volume



Hall probe

A device used to measure magnetic flux density.

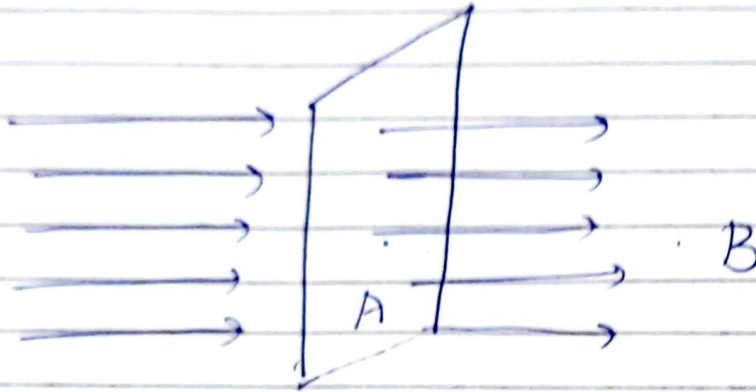


Physics With Yameen

7/11/17

Electro magnetic Induction

Magnetic flux



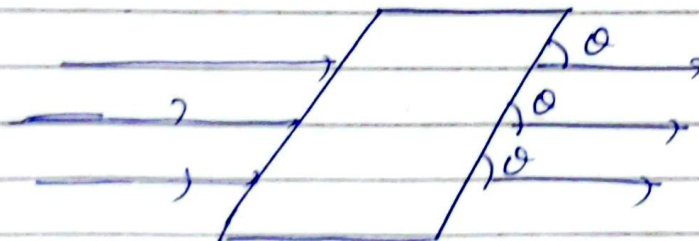
B: magnetic flux density

A: Area held perpendicular

$$\Phi = BA$$

Φ : magnetic flux

product of magnetic flux density and area held perpendicular to it



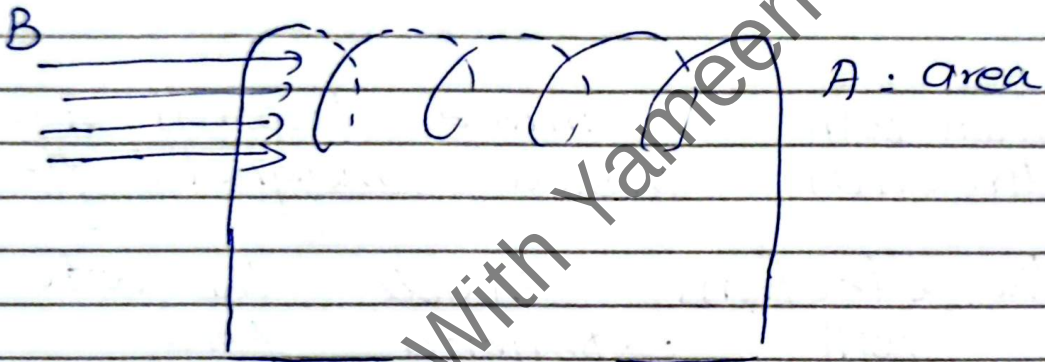
$$\Phi = BA \sin \theta$$

unit of flux

Weber (Wb)

$$Wb = T m^2$$

Magnetic flux Linkage



NBA

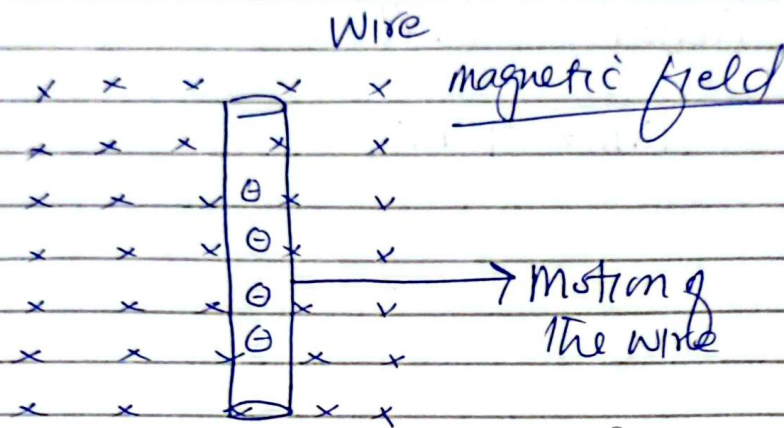
is known as

flux linkage

$$N\Phi = \text{flux Linkage}$$

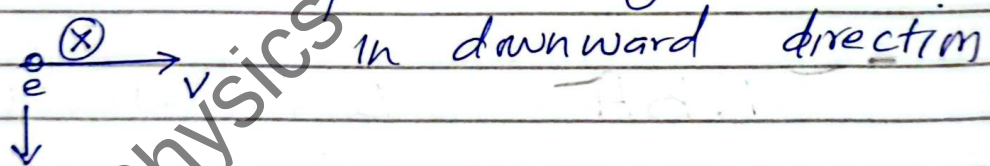
↓
number of turns

Faraday's Law



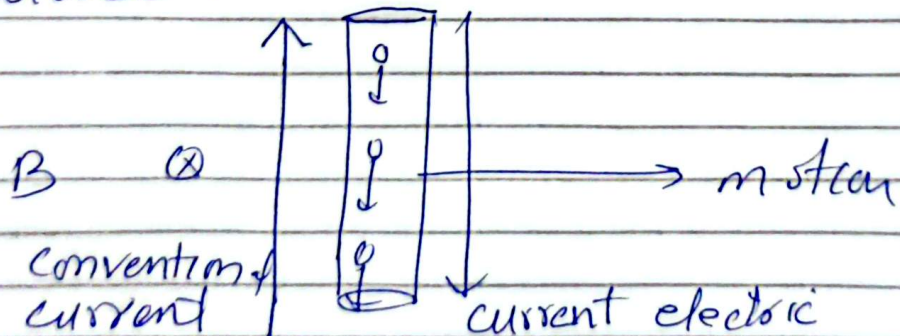
When wire is moved towards right direction inside magnetic field

The electrons inside the wire will experience force

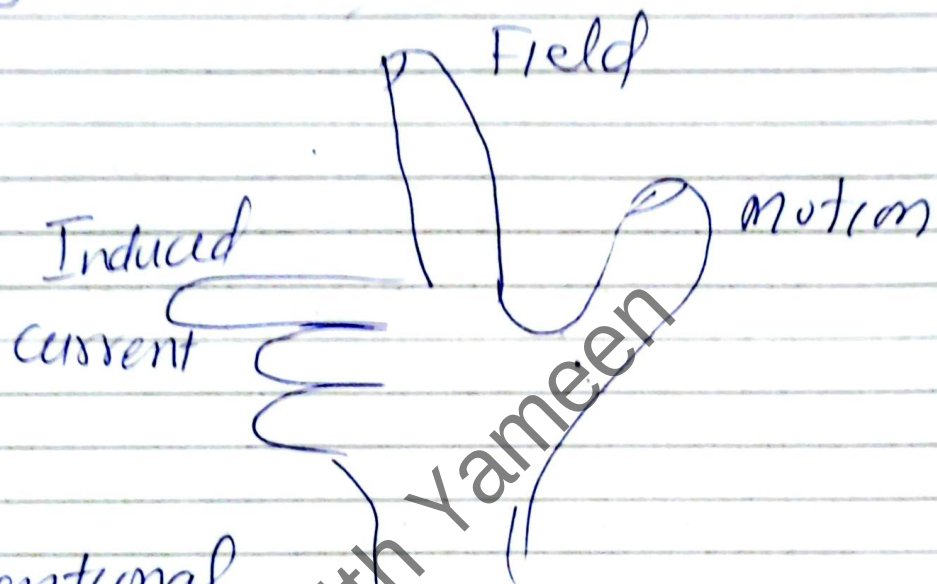


So electrons will start moving current will

Induce.



Direction of Induced current
is determined using
Right hand rule.

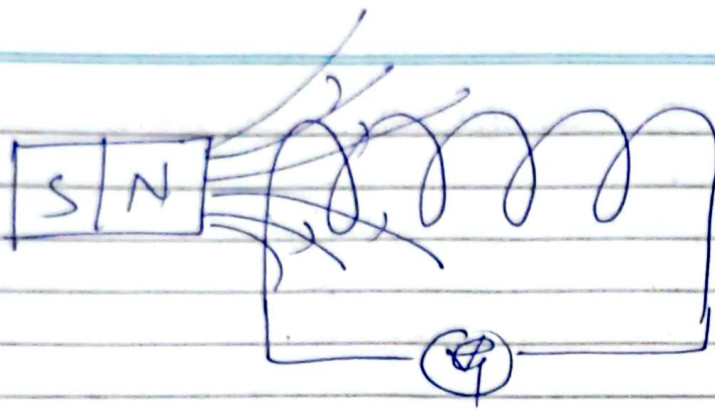


I: conventional
current

Faraday Law of electromagnetic
Induction.

Induced emf is directly
proportional to rate of
change of magnetic flux
Linkage

$$\mathcal{E} = \frac{\Delta(N\Phi)}{\Delta t}$$



G: Galvanometer

Factors effecting induced emf

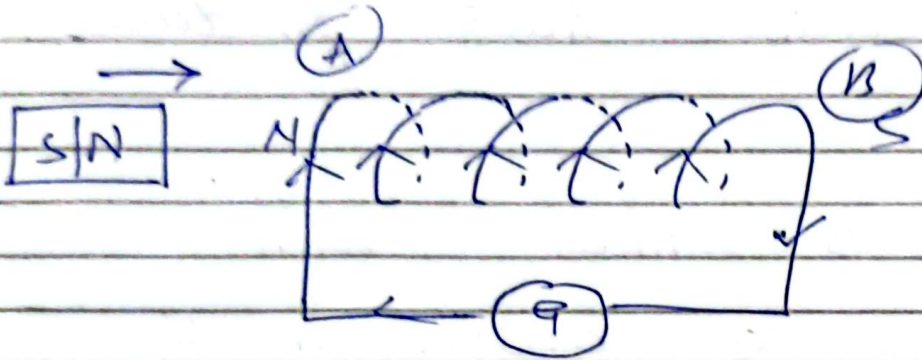
* If speed of magnet is increased, emf increases

* If magnet is moved towards the coil, deflection of galvanometer is on one side. If it is moved away, deflection is on other side.

* If magnet and coil are stationary, then no induced emf.

* emf depends on relative motion of coil and magnet.

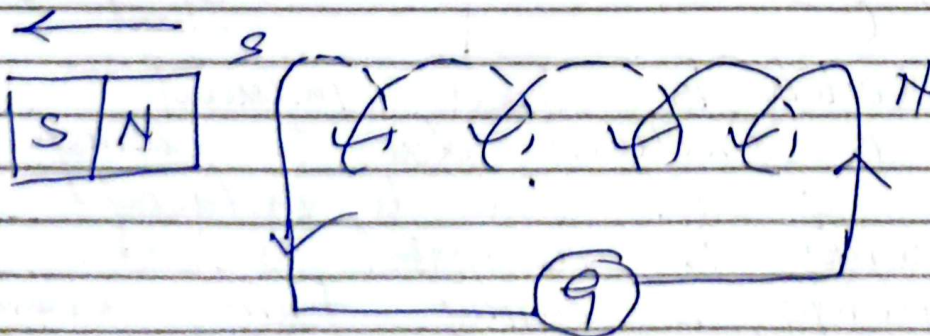
Direction of induced emf.



When North pole is moved toward the coil, emf is induced due to induced emf. Current is generated so magnetic field is generated in coil.

This magnetic field opposes the magnetic field of magnet.

So North pole is induced on side A and south of side B.



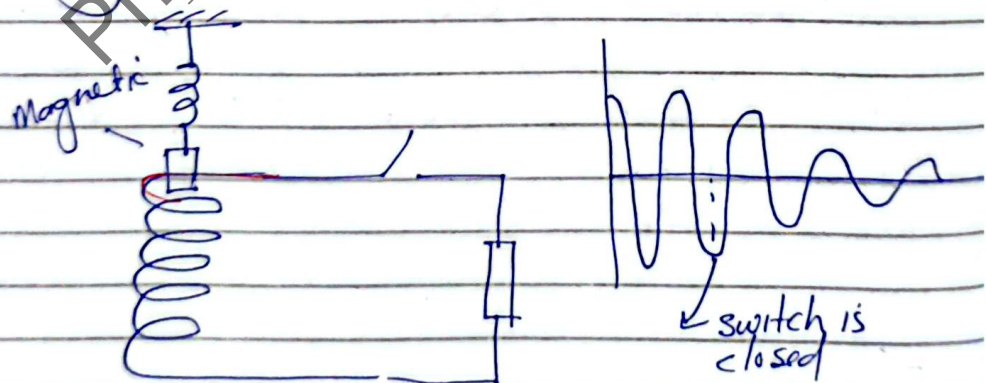
Lenz's law

Direction of induced current is such that it opposes the cause that produces it.

this is linked with law of conservation of energy

$$\mathcal{E} = - \frac{\Delta(N\Phi)}{\Delta t}$$

—: Sign was introduced by Lenz's

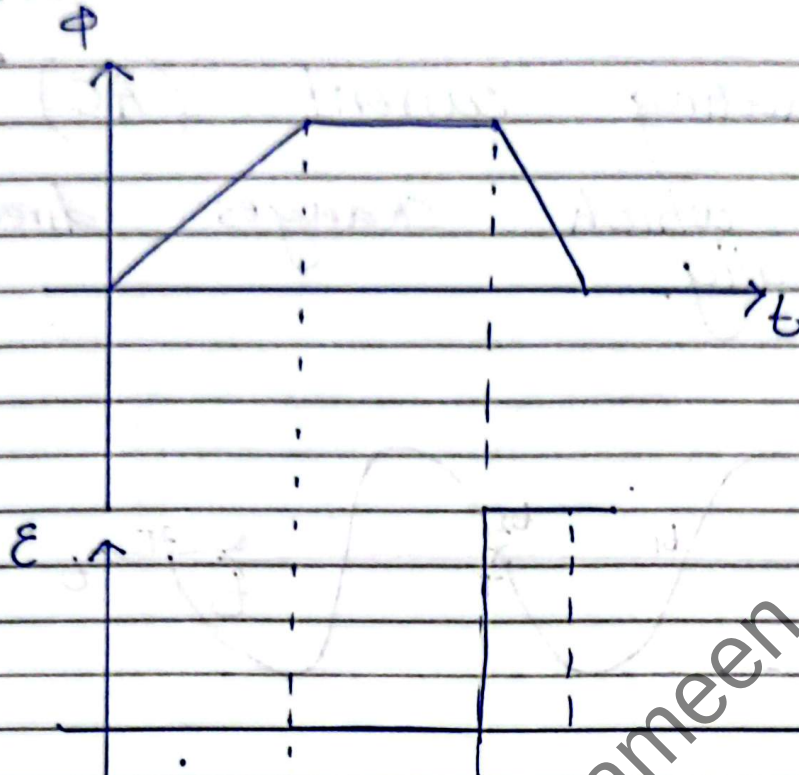


when switch is closed induced current is produced due to change in flux of coil. Due to induced current energy is dissipated as heat. this energy is coming from energy of magnet. so damping is caused

$$\mathcal{E} = - \frac{\Delta(N\Phi)}{\Delta t}$$

$$\mathcal{E} = -N \left(\frac{\Delta\Phi}{\Delta t} \right)$$

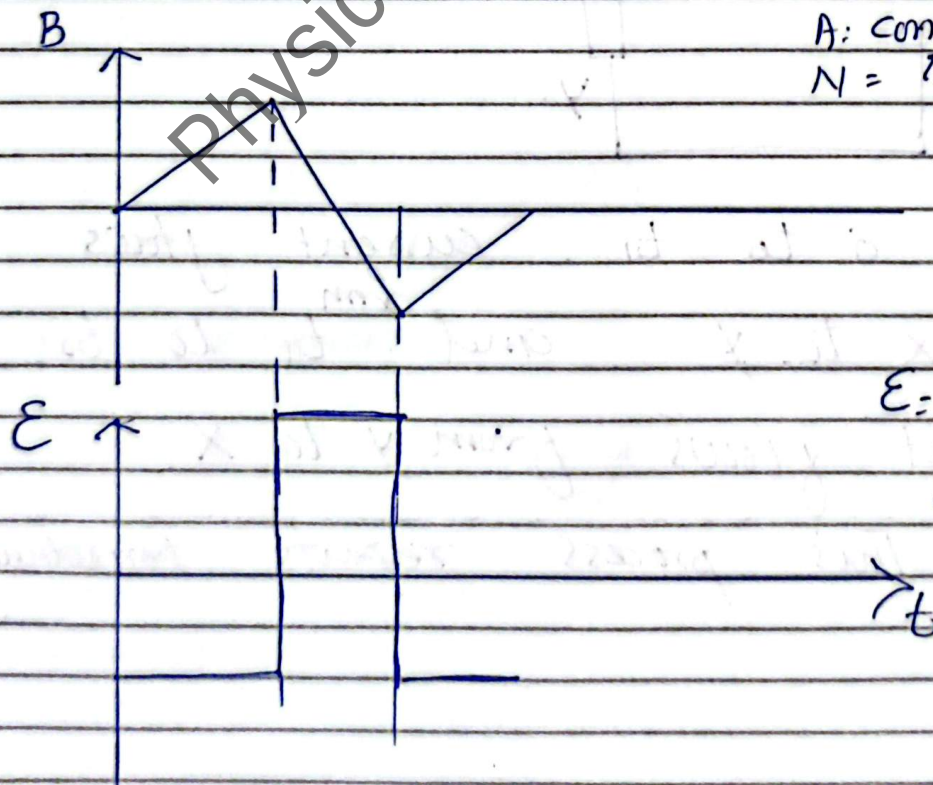
$$\mathcal{E} = -N(g\omega)$$



$$\Phi = BA$$

A: constant

$N = 1$



$$\mathcal{E} = - \frac{\Delta(N\Phi)}{\Delta t}$$

$$\mathcal{E} = -NA \frac{\Delta B}{\Delta t}$$

$$\mathcal{E} \propto -g\omega$$