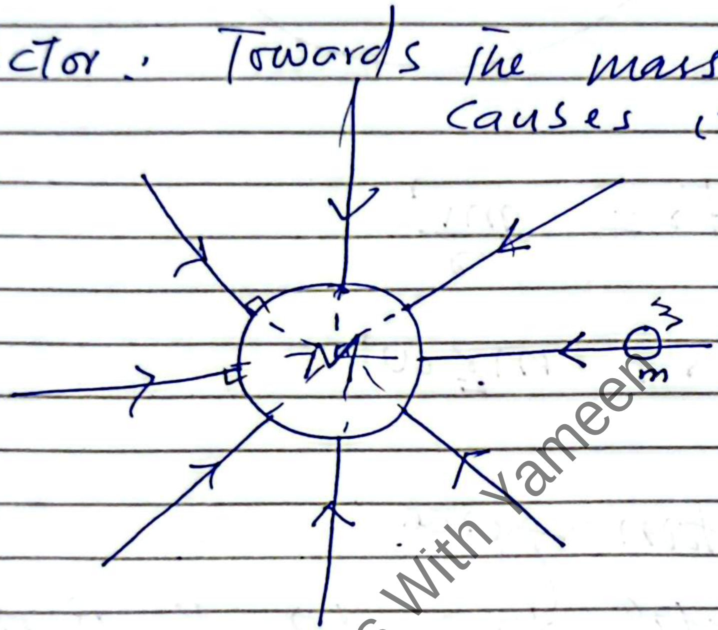
Gravitational Fields

Define gravitational field

Force per unit mass

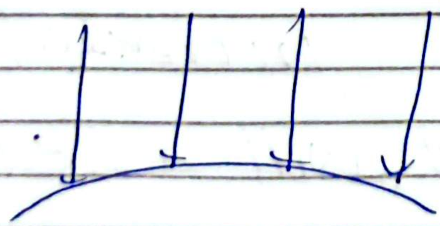
$g = \frac{F}{m}$ $\quad N\,kg^{-1}$

Vector: Towards the mass that causes it.



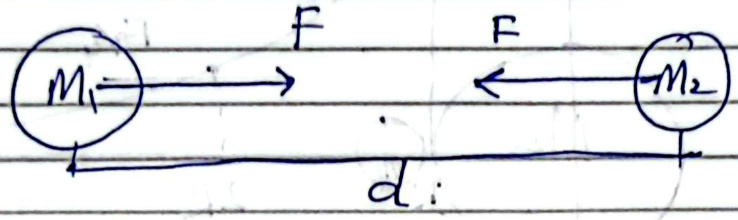
Field is non uniform as field lines are not parallel.

Near to the surface of earth



field lines are parallel which show field is uniform near to the surface of earth.

Newton's Law of gravitation



<u>Define</u>

"Force between two point masses is directly prop to product of masses and inversely proportional to square of distance between their centers"

$$F \propto M_1 M_2$$

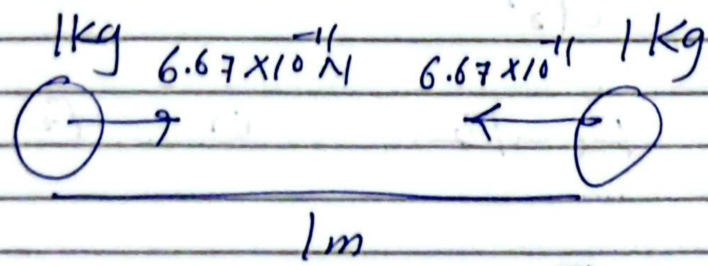
$$F \propto \frac{1}{d^2}$$

$$F \propto \frac{M_1 M_2}{d^2}$$

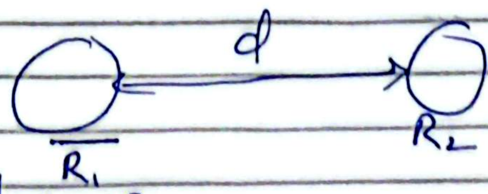
$$\boxed{F = G\frac{M_1 M_2}{d^2}}$$

G: universal gravitational constant

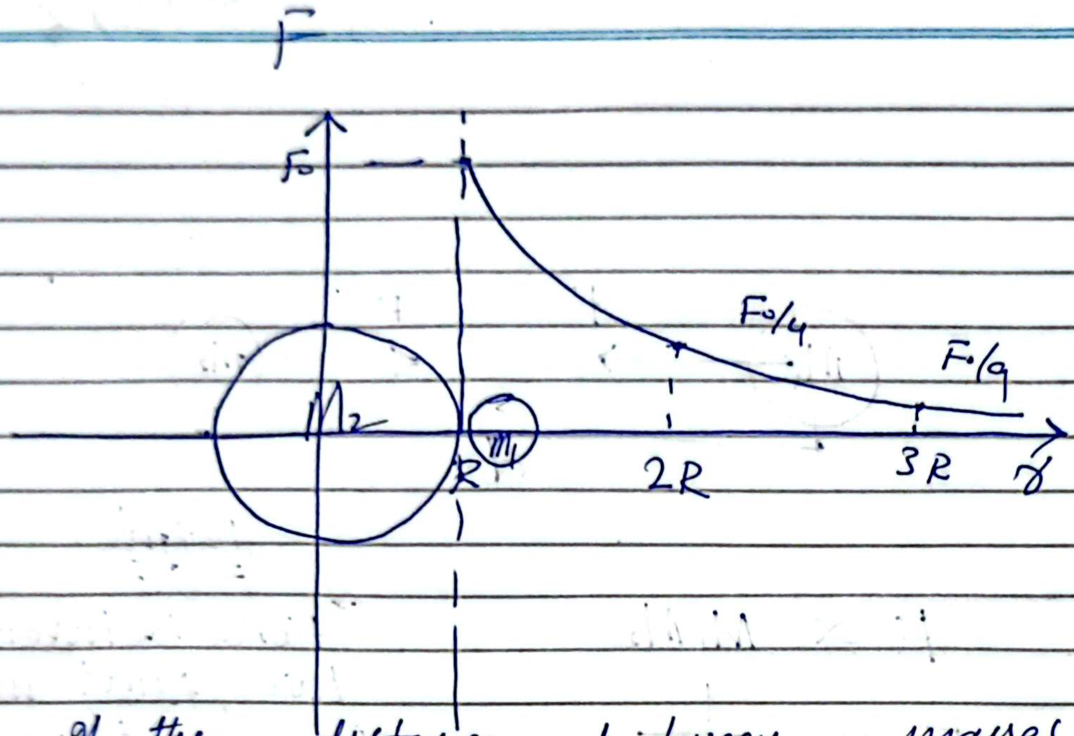
G = $6.67 \times 10^{-11}\ N\,m^2/kg^2$



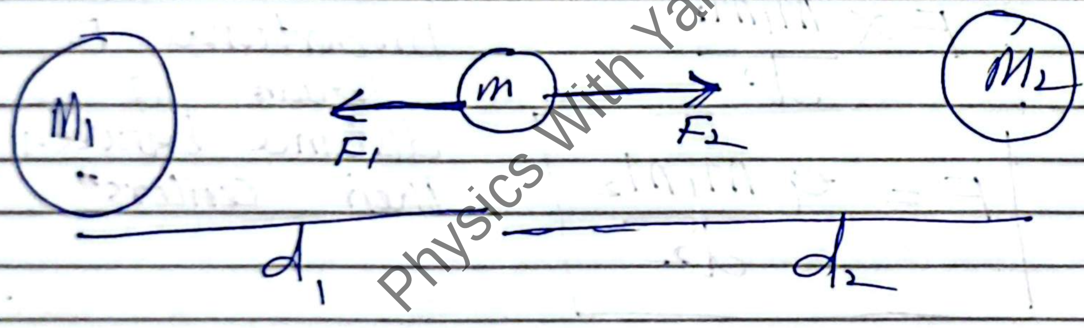
<u>point mass:</u>



If $d \gg R_1$ or $d \gg R_2$



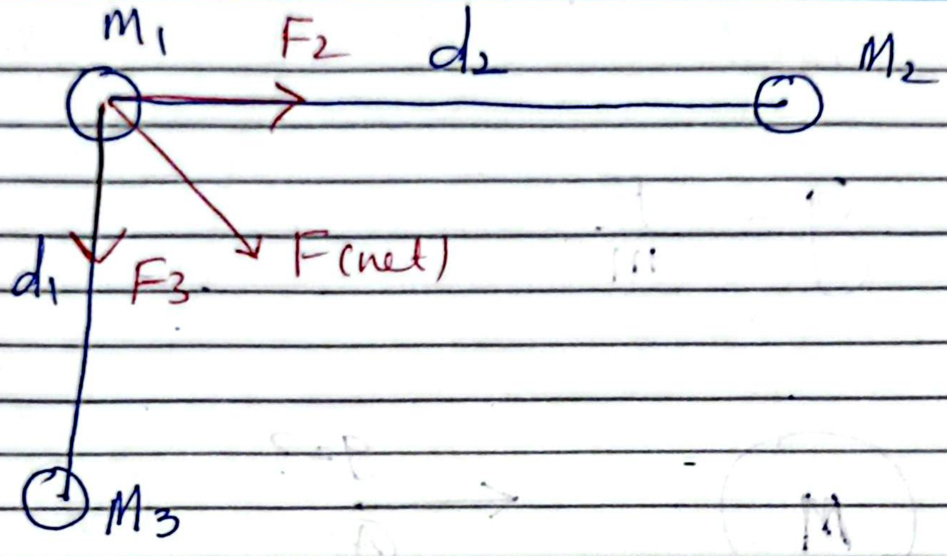
If the distance between masses is increased then force will decrease.



$$F_1 = \frac{G M_1 m}{d_1^2}$$

$$F_2 = \frac{G M_2 m}{d_2^2}$$

$$F_{net} = F_1 - F_2 \quad \text{or} \quad F_2 - F_1$$

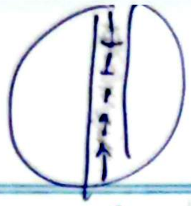


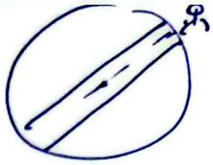
find total force on M_1

$$F_2 = \frac{G M_1 M_2}{d_2^2}$$

$$F_3 = \frac{G M_1 M_3}{d_1^2}$$

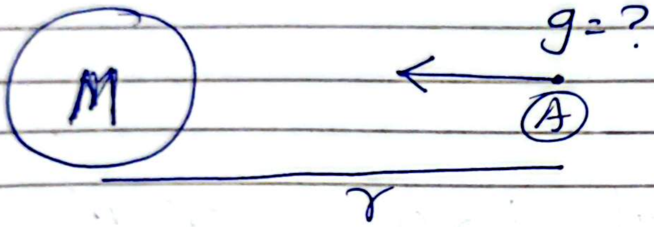
use vector addition rules to find net force.





Gravitational field strength

$$g = \frac{F}{m}$$



Put mass m at A and find force (F)

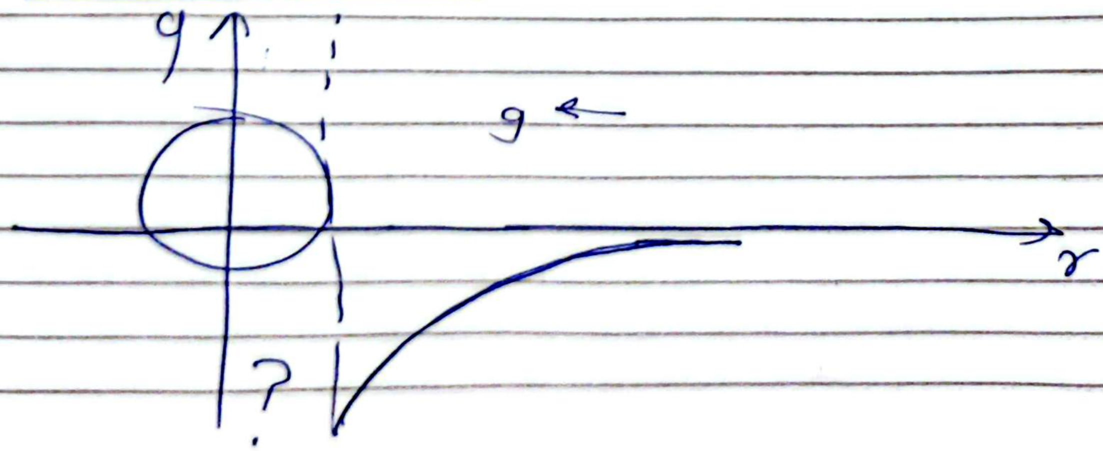
$$F = \frac{GMm}{r^2}$$

Now use $g = \frac{F}{m}$

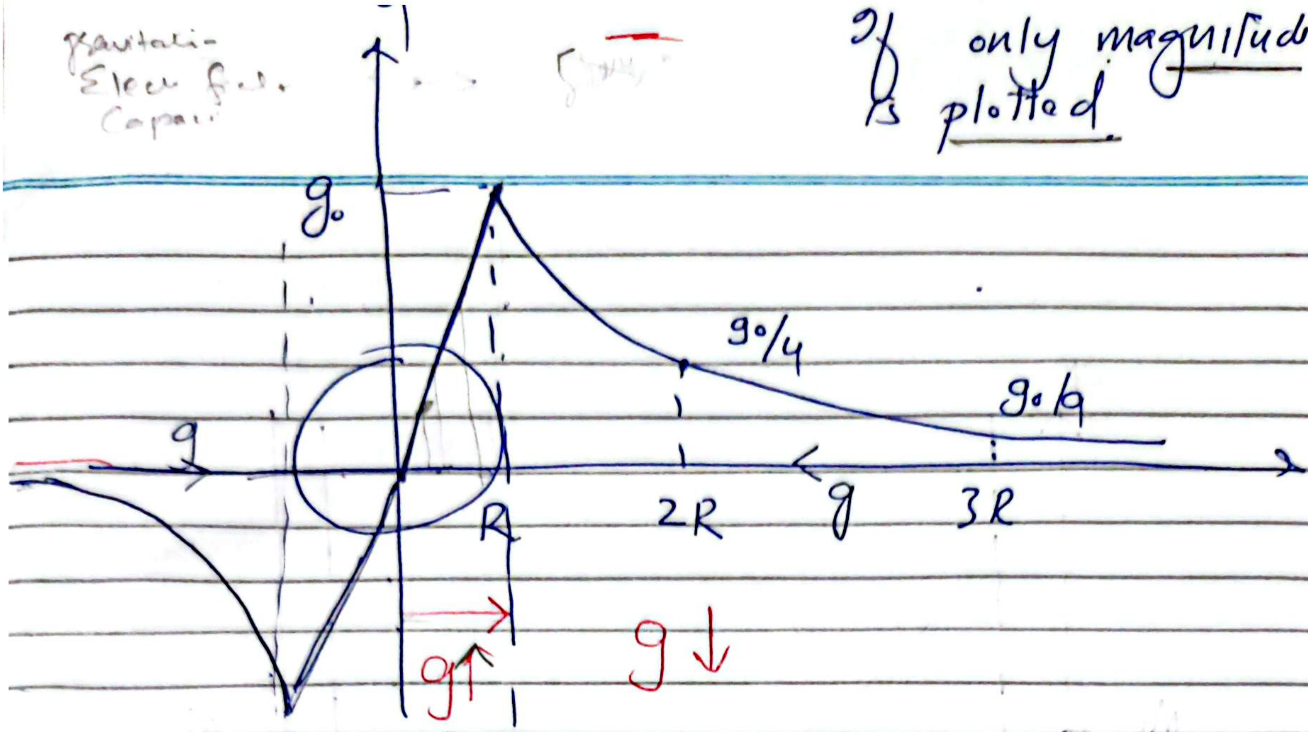
$$g = \frac{GMm}{r^2} / m$$

$$\boxed{g = \frac{GM}{r^2}}$$

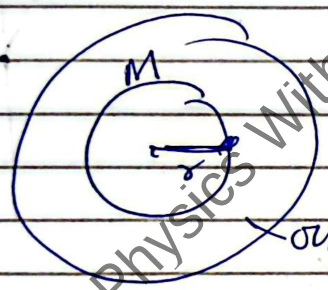
[only applicable away from surface $r \geq R$



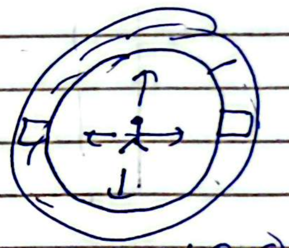
If only magnitude is plotted



g inside The planet



outer mass will not effec



net $g=0$

$$g = \frac{GM}{r^2}$$

$$V = \frac{4}{3}\pi r^3$$

$$g = \frac{G\rho\frac{4}{3}\pi r^3}{r^2}$$

$$\rho = \frac{M}{V}$$

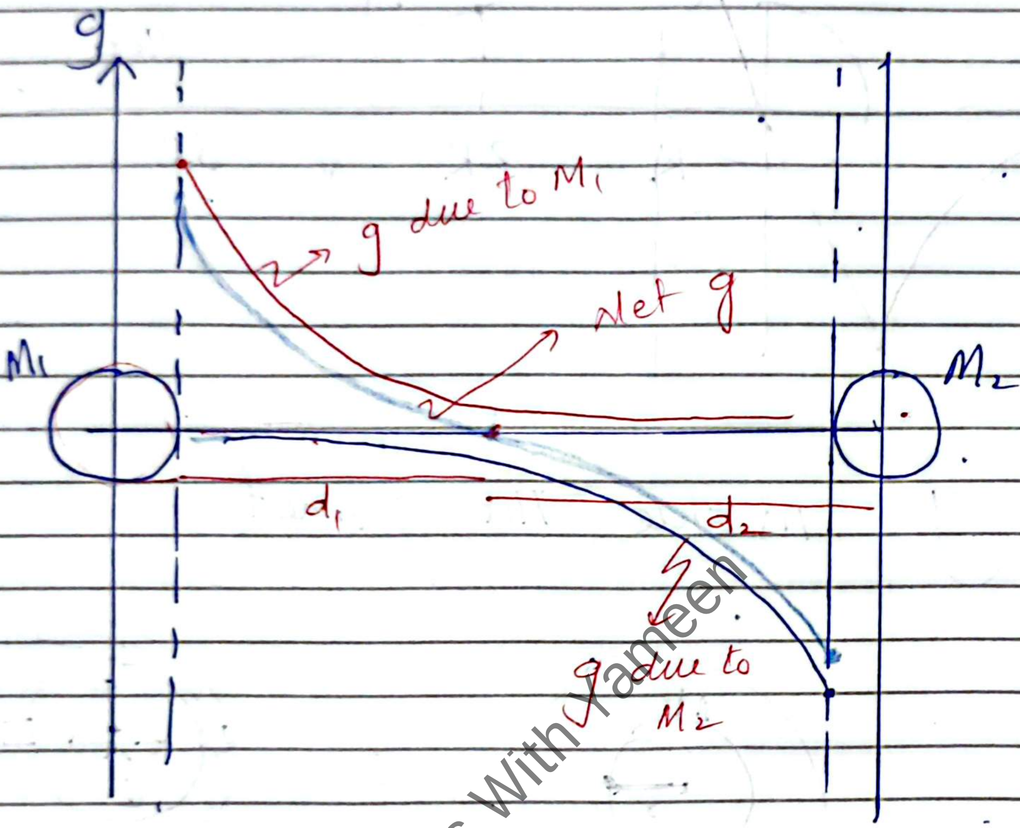
$$M = \rho V$$

$$M = \rho\frac{4}{3}\pi r^3$$

$$g = \boxed{G\rho\frac{4}{3}\pi}\, r$$

$$g \propto r$$

Gravitational field strength between two masses.



$$\frac{\cancel{G}M_1}{d_1^2} = \frac{\cancel{G}M_2}{d_2^2}$$

$$\frac{M_1}{M_2} = \frac{d_1^2}{d_2^2}$$

$$\frac{d_1}{d_2} = \sqrt{\frac{M_1}{M_2}}$$

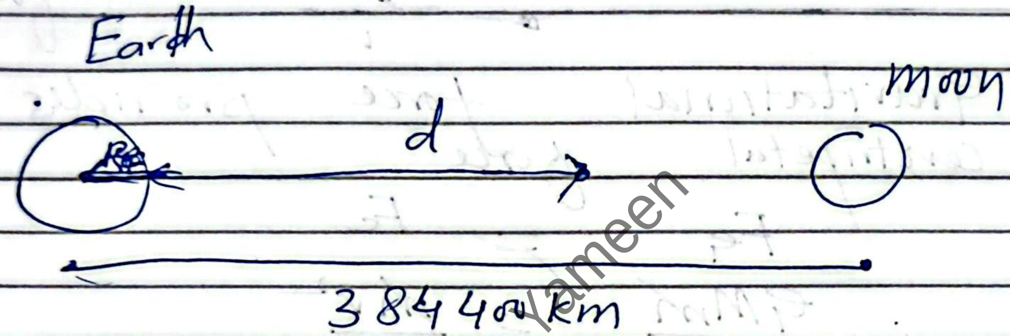
If $M_1 = M_2$ $\quad d_1 = d_2$

If $M_1 > M_2$ $\quad d_1 > d_2$

The point where g is zero

is closer small mass.

Find distance from center surface of earth where g is zero due to moon and earth.



$$\frac{GM_E}{(d\cancel{+R_E})^2} = \frac{GM_M}{(384400-d)^2}$$

$$\frac{\cancel{G}M_E}{d^2} = \frac{\cancel{G}M_M}{(384400-d)^2}$$

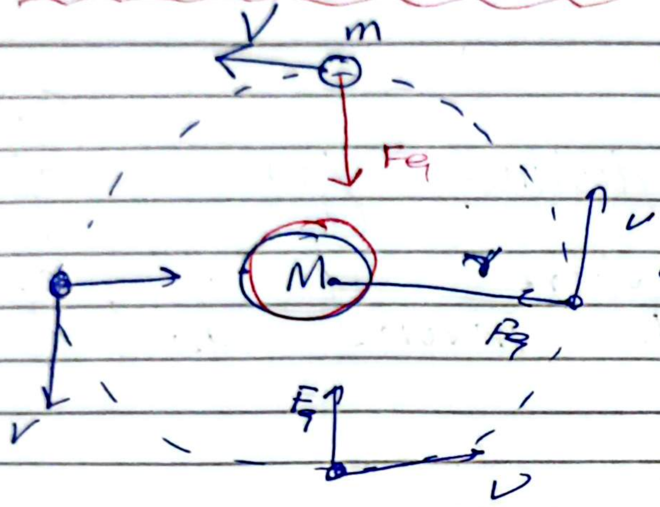
$$\frac{(384400-d)^2}{d^2} = \frac{M_M}{M_E}$$

$$\frac{384400-d}{d} = \sqrt{\frac{M_M}{M_E}}$$

$$\frac{384400-d}{d} = \sqrt{\frac{7.34\times10^{22}}{6\times10^{24}}}$$

$$\frac{384400-d}{d} = 0.11 \qquad d = 346306\ \text{km}$$

Circular orbits



→ m is orbiting around M

• centripetal force is provided by gravitation force

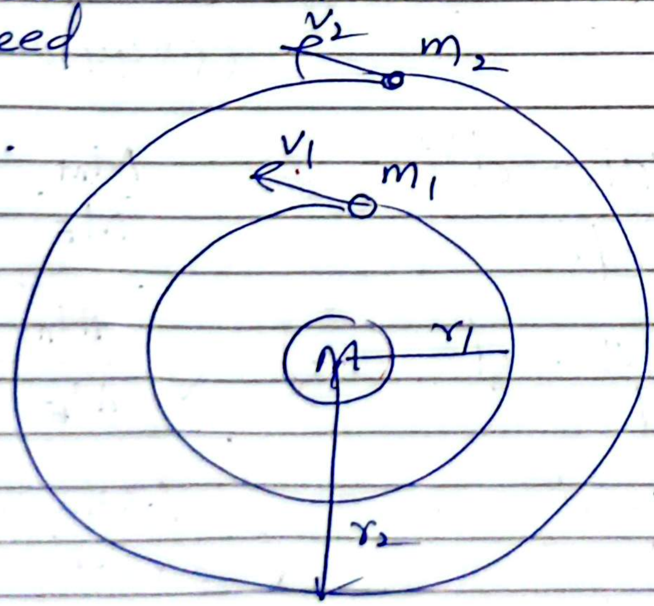
Gravitational force provides centripetal force

$$F_g = F_c$$

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\therefore V = \sqrt{\frac{GM}{r}}$$

This is known as orbital speed



$$\boxed{\frac{V_1}{V_2} = \sqrt{\frac{r_2}{r_1}}}$$

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

Relation of ω

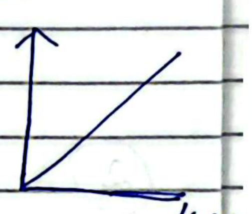
$$\frac{GMm}{r^2} = m r \omega^2$$

$\cancel{\omega^2 = \frac{1}{GM}\left[r^3\right]}$ $GM = r^3 \omega^2 \rightarrow (1)$

$r^3 \omega^2 =$ constant

Kepler's Third Law

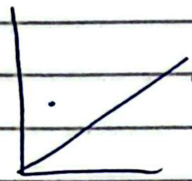
$\omega^2 = \frac{GM}{r^3}$



for time period

$\omega = \frac{2\pi}{T}$ put (1)

$\omega = \frac{\theta}{t}$



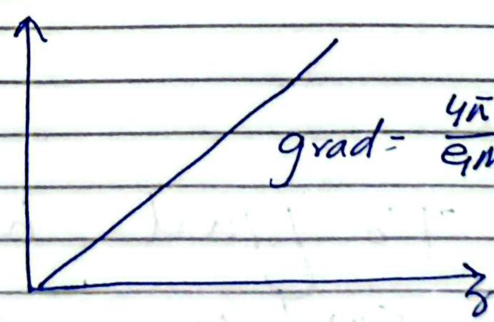
$GM = r^3 \left(\frac{2\pi}{T}\right)^2$

$GM = \frac{r^3 4\pi^2}{T^2}$

$\frac{T^2}{r^3}$ = consta

$$\underbrace{\boxed{T^2}}_{y} = \frac{4\pi^2}{GM} \underbrace{\boxed{r^3}}_{x}$$

$T^2 \propto r^3$



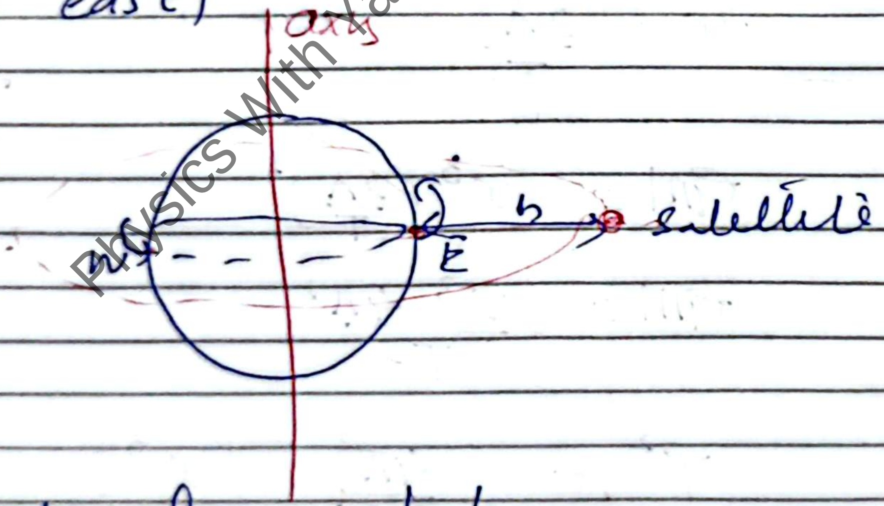
Geostationary Earth Satellite

Geostationary sattellite stays at the same position relative to the earth

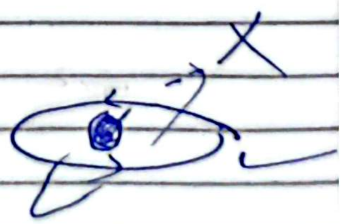
It has following properties

① Time period = 24 hrs (same as Earth about its axis)

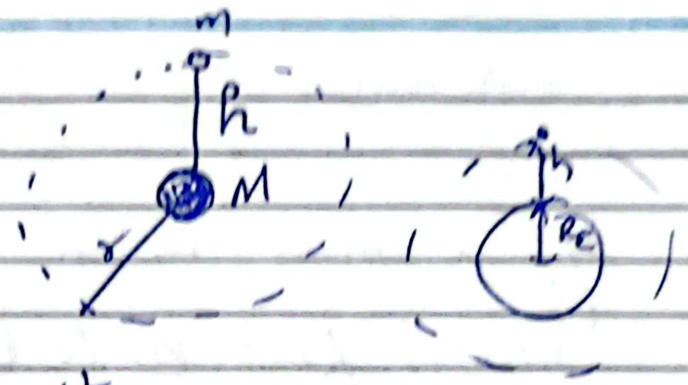
② Direction of rotation is same as the earth [from West to east]



③ Equatorial orbit



To find height from the surface of earth



$$\frac{GMm}{r^2} = mr\omega^2$$

r is distance between earth and moon

$$GM = r^3\left(\frac{2\pi}{T}\right)^2$$

$$r^3 = \frac{GM}{4\pi^2}\,T^2$$

$$r^3 = \frac{G \times 6\times10^{24}}{4\pi^2} \times (24\times3600)^2$$

$$r^3 = \frac{6.67\times10^{-11} \times 6\times10^{24}}{4\pi^2} \times (24\times3600)^2$$

$$r^3 = 7.56\times10^{22}\ m^3$$

$R_E = 6378$ km

$r = 42283792$ m

42283792

$h + R_E = \cancel{4229639}$

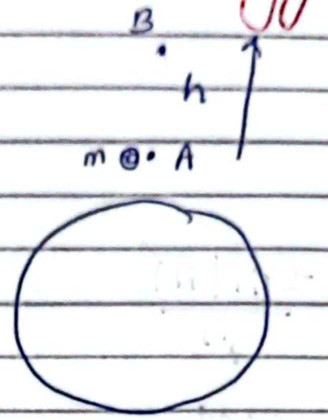
$h = \cancel{4229639} - R_E$

$h = \cancel{4229639} - 6378000$

$= 4228379 - 6378000$

$h = 35905792$ m $= 35905.792$ km

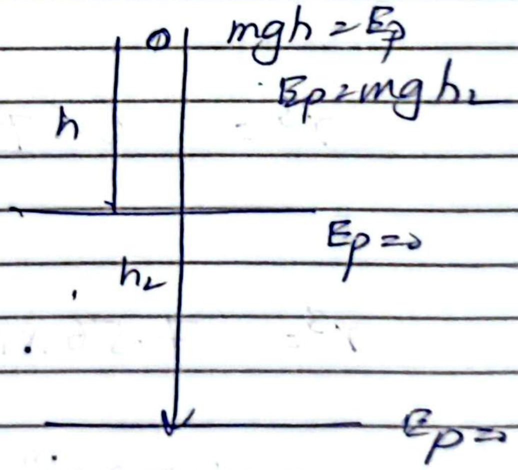
Gravitational potential energy



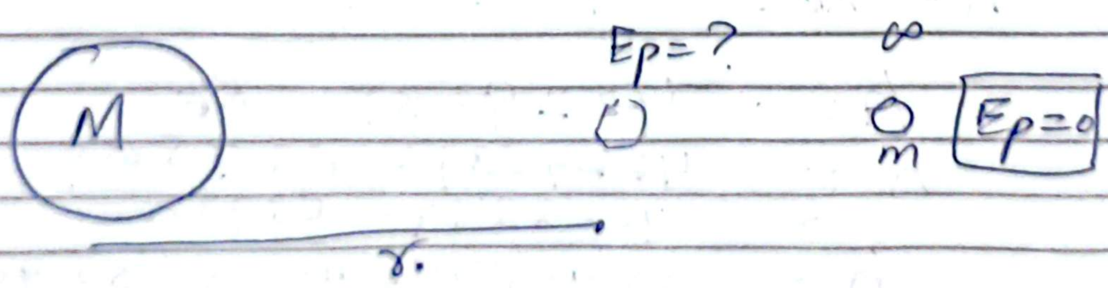
If a mass $\underline{m}$ is moved from (A) to (B) through height (h)

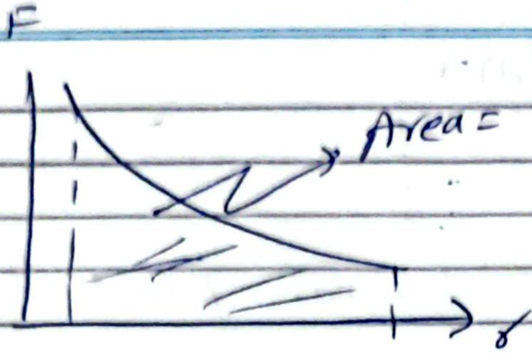
$\Delta E_p = mgh$

This is only valid when h is small so that g is not changing b/w A and B.



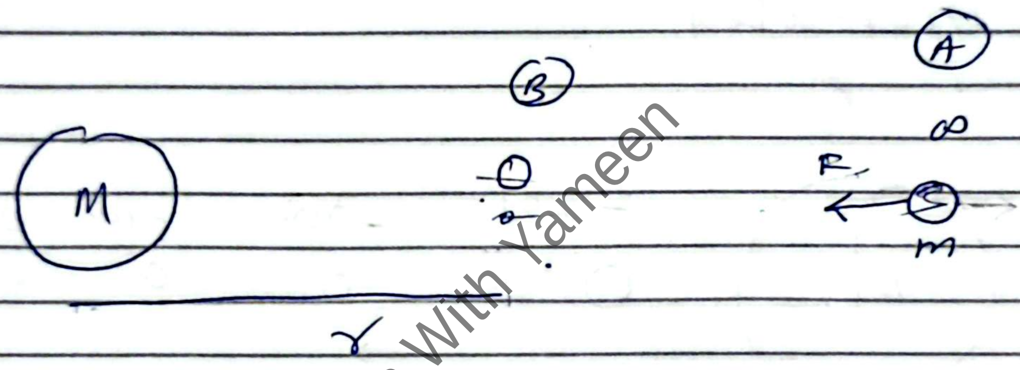
Now in A2 we will take reference point at infinity (∞).





* As Force is not constant so $W = Fd$ is not applicable

We will take area to ~~the~~ find work done



Work done to move object m from infinity to point B

$$F = \frac{GMm}{r^2}$$

$$W = \int_{\infty}^{r} \frac{GMm}{r^2} \, dr$$

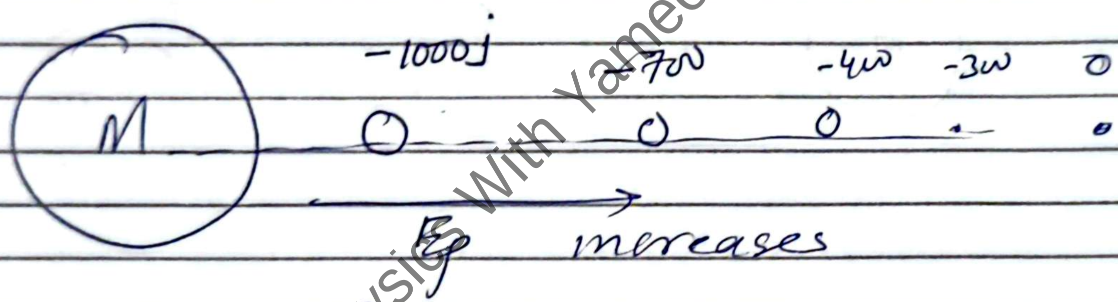
$$\boxed{E_p = -\frac{GMm}{r}}$$

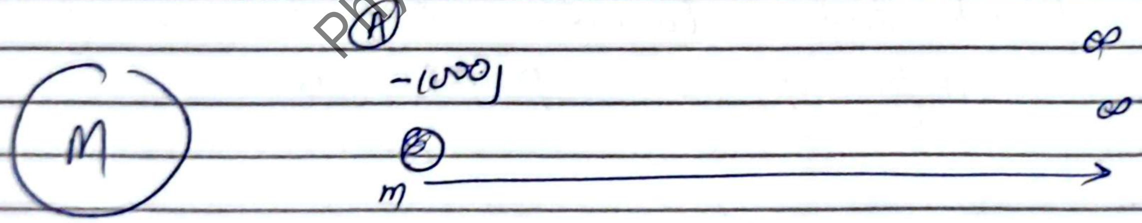
$$\boxed{E_p = \frac{-GMm}{r}}$$

E_p is gravitational potential energy

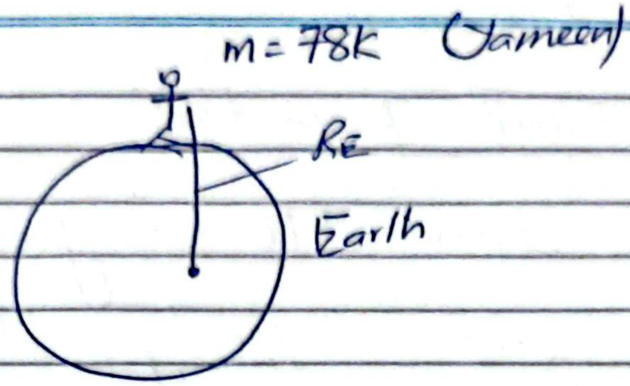
$E_p = 0$ at ∞

E_p will increase as r is increased





-1000j p.e means 1000j of energy is required to move m from A to ∞



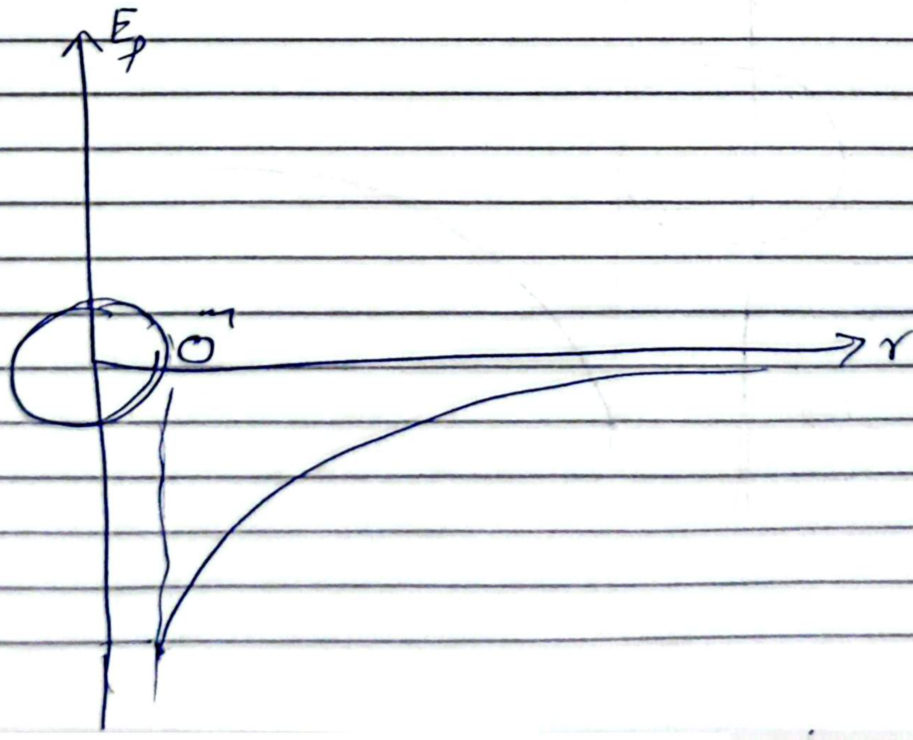
find Gravitational potential energy of sir Yameen at the surface of earth.

$R_E = 6400\,km$

$$E_p = -\frac{GMm}{r}$$

$$= \frac{-6.67\times10^{-11}\times 6\times10^{24}\times 78}{6400\times10^{3}\,m}$$

$$E_p = -4\times10^{9}\,J$$



ϕ

Gravitational potential

Work done to bring a unit test mass from infinity to a point inside gravitational field is known as gravitational potential.

$$E_p = -\frac{GMm}{r}$$

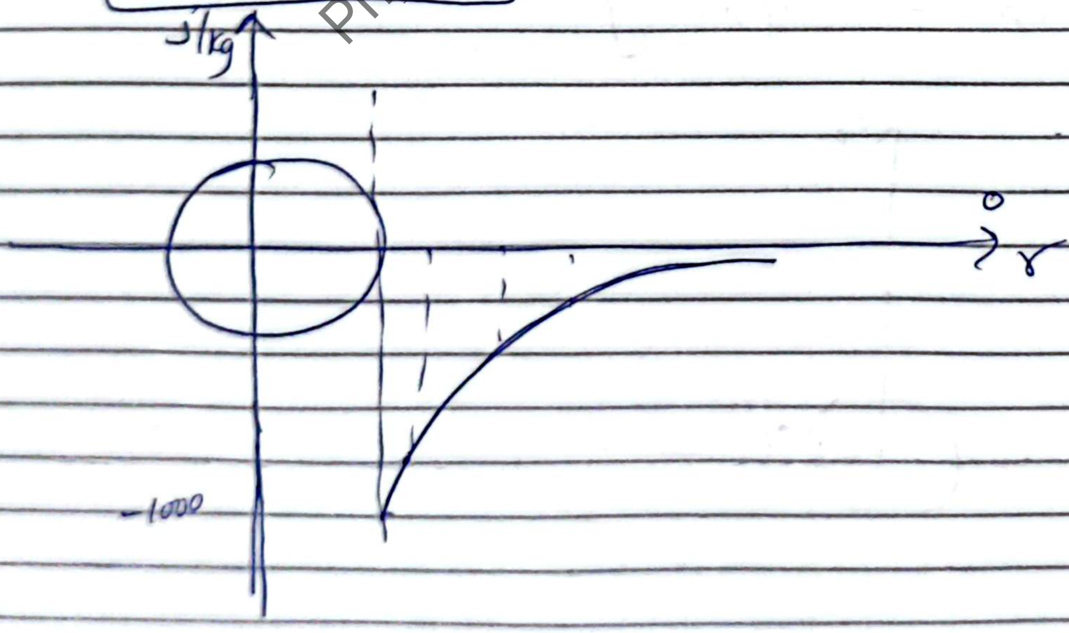
$$\phi = \frac{W}{m}$$

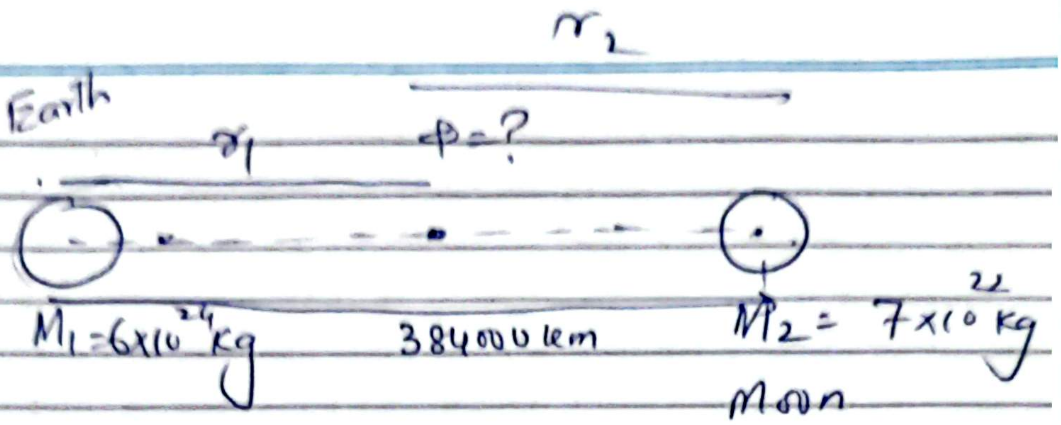
$$\phi = \frac{E_p}{m}$$

ϕ = J/kg unit

scaler

$$\boxed{\phi = -\frac{GM}{r}}$$





$R_E = 6400$ km

~~total~~ b/w

find potential at mid point of earth and moon.

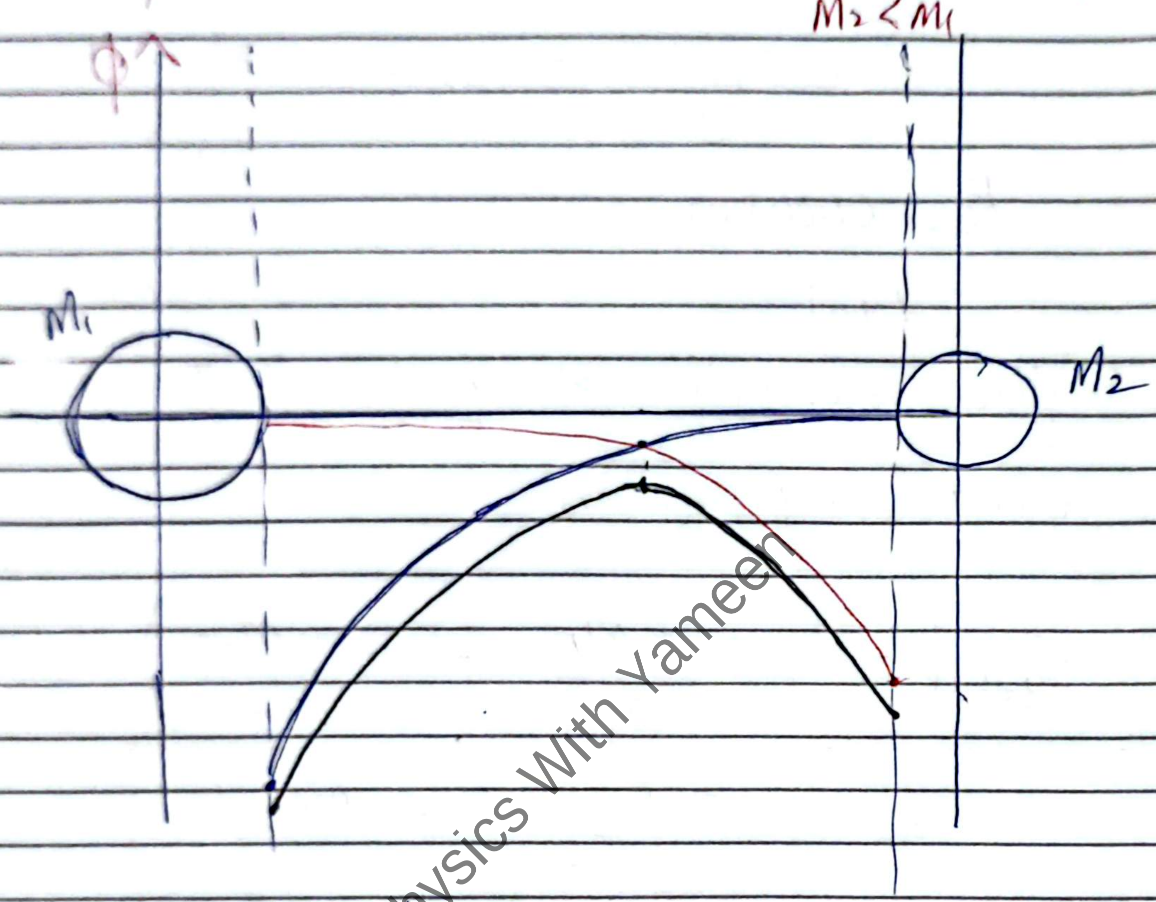
$$\phi_{total} = -\frac{GM_E}{r_1} + \left(-\frac{GM_M}{r_2}\right)$$

$$= -G\left[\frac{M_E}{r_1} + \frac{M_M}{r_2}\right]$$

$$= -6.67\times10^{-11}\left[\frac{6\times10^{24}}{192000\times10^{3}} + \frac{7\times10^{22}}{192000\times10^{3}}\right]$$

$\phi = -\frac{GM}{r}$ ✓ $g = \frac{GM}{r^2}$

Graphs of Net gravitational potential b/w two masses



—: Due to M_1

—: Due to M_2

—: Net ϕ

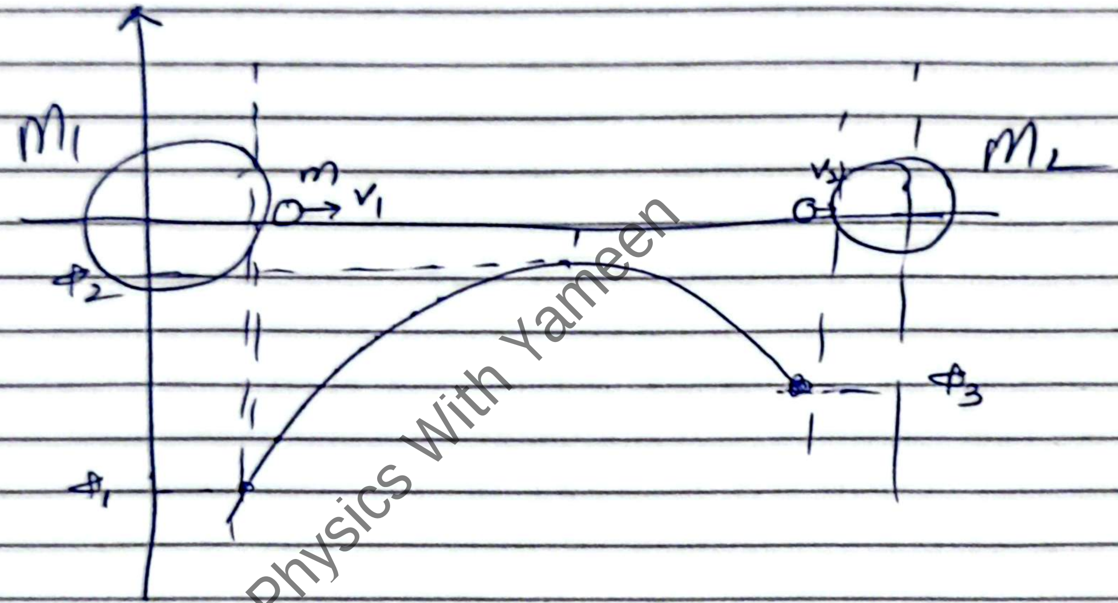
Relation b/w gravitational field strength and gravitational potential

$g = \text{gradient}(\phi)$

Relation b/w potential and potential energy

$$E_p = m\,\Phi$$

$$\Delta E_p = m\,\Delta\Phi$$



m kg moves from surface of M_1 to the surface of M_2

$$\Delta E_p = m(\Phi_3 - \Phi_1)$$

$$\Delta E_p + \Delta E_k = 0$$

Law of conservation of energy

$$\Delta E_k = -\Delta E_p$$

$$\Delta E_k(max) = m(\Phi_2 - \Phi_1)$$