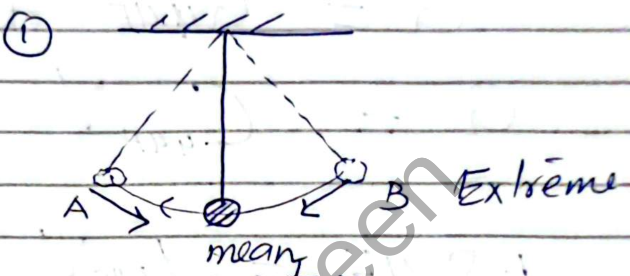
Oscillation

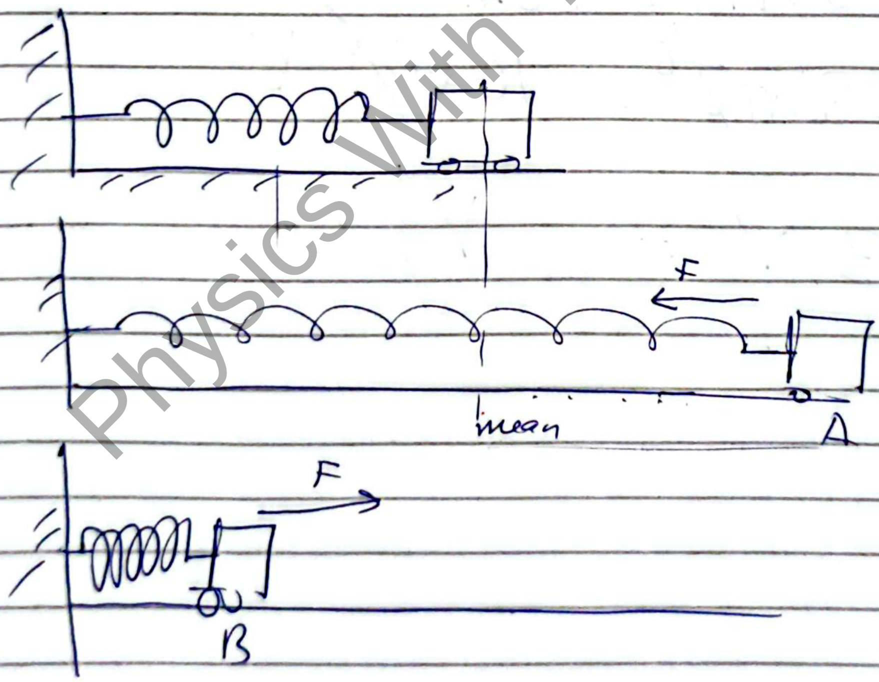
Oscillation

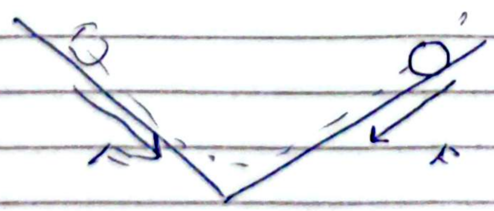
"Periodic motion between two ~~fixed~~ points"

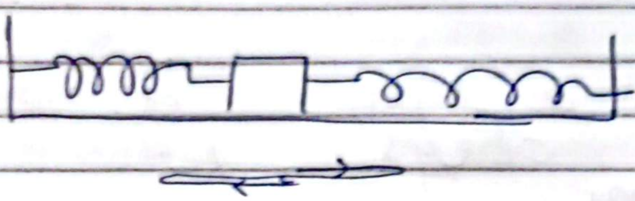
Example

1. 

A and B are two point

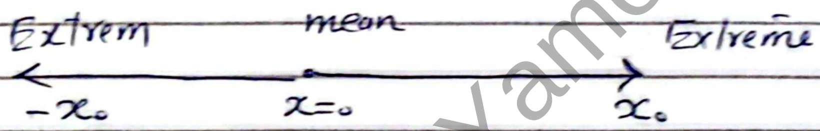
2. 

3. 

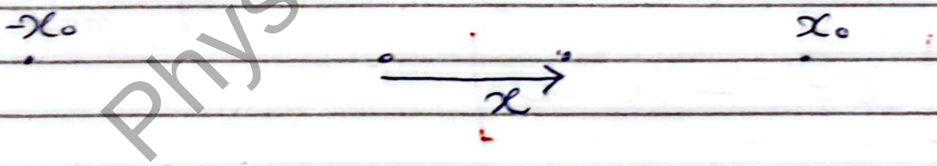


Amplitude

"Maximum displacement from mean position"

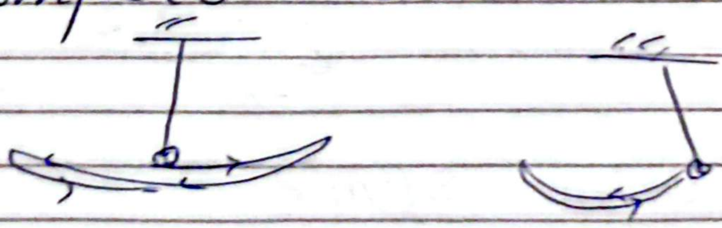


Displacement: Shortest distance from mean position



Time period

time to complete one oscillatio



Frequency: Number of cycles (oscillations) per unit time.

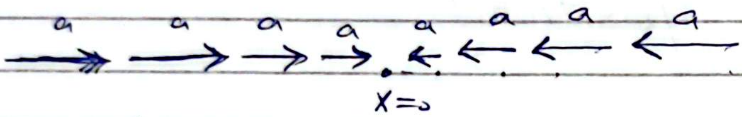
$f = \frac{1}{T}$ $\omega = \frac{2\pi}{T}$

$\boxed{\omega = 2\pi f}$

Simple Harmonic Motion (SHM)

This is a type of oscillation which obeys following conditions

i) Acceleration is directly proportional to displacement mean position



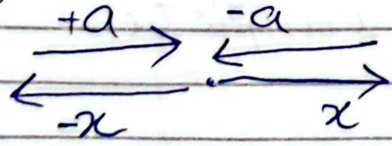
$a \propto x$

ii) acceleration is directed towards the mean position

General Equation of SHM

$\boxed{a = -\omega^2 x}$

(–) sign shows a is opposite to displacement

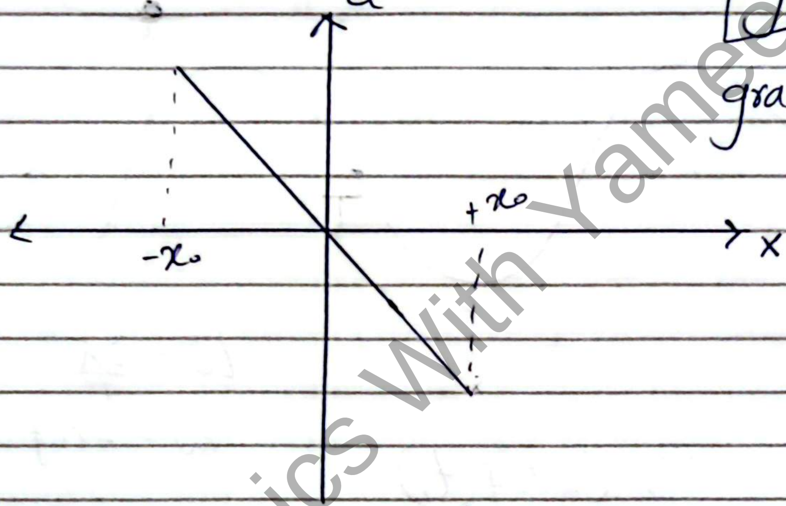


Define SHM?

"Motion in which acceleration is directly proportional to displacement from mean position and directed towards mean position"

Graph of a and x

$$\boxed{a} = -\omega^2 \boxed{x}$$
$$\boxed{y} = m \boxed{x}$$
$$\text{grad} = -\omega^2$$



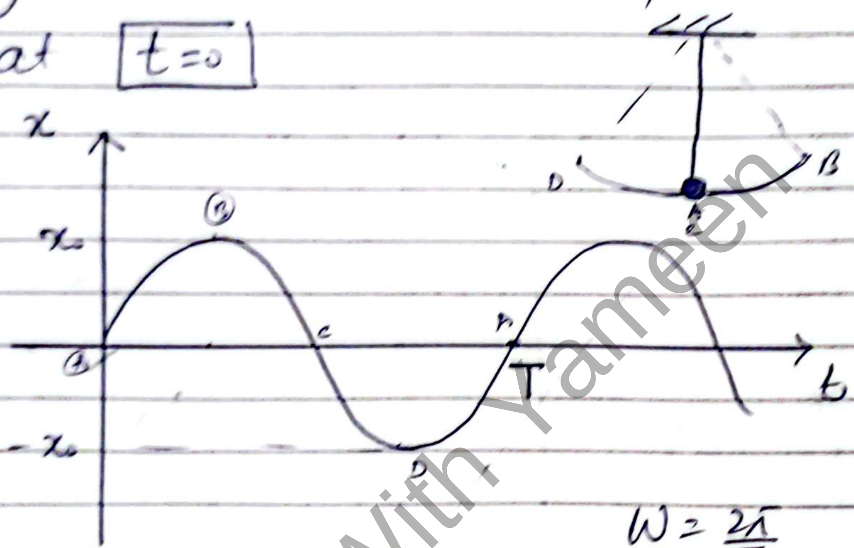
Maximum acceleration

$a_{max} = -\omega^2 x_0$ when x is max

Equations for displacement

$x = x_0 \sin(\omega t)$ This equation satisfies $a = -\omega^2 x$

This is applicable when object is at mean position at $\boxed{t=0}$



$\omega = \frac{2\pi}{T}$

$\omega = 2\pi f$

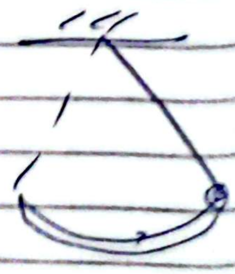
$X = X_0 \sin(2\pi f t)$

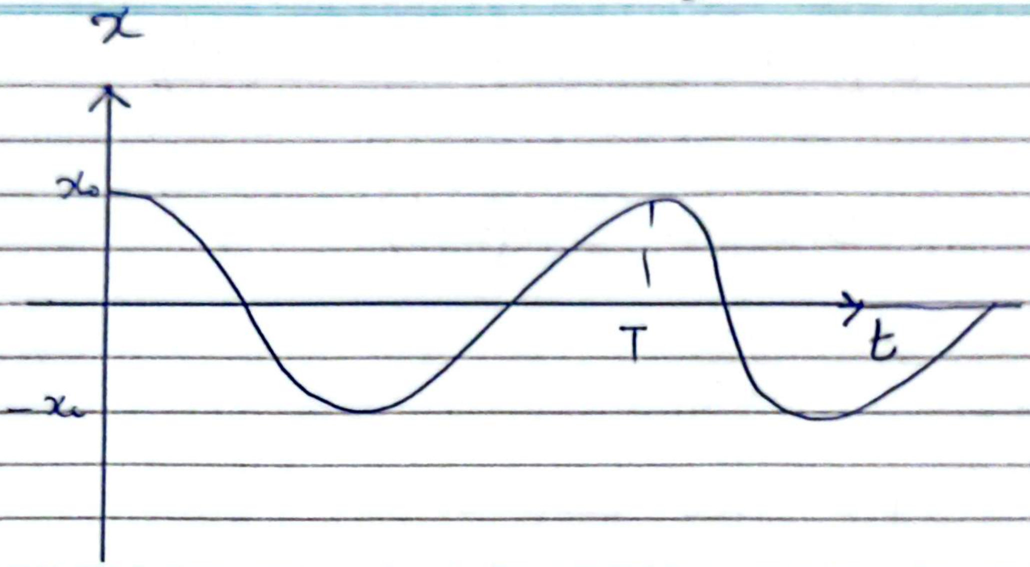
$X = X_0 \sin\left(\frac{2\pi}{T} t\right)$ unit of t and T same

<u>other Equation</u>

$X = X_0 \cos(\omega t)$

at $t=0$ $X = X_0$





an object starts from mean position at $t=0$ it has amplitude of 10 cm and time period of 0.4 s

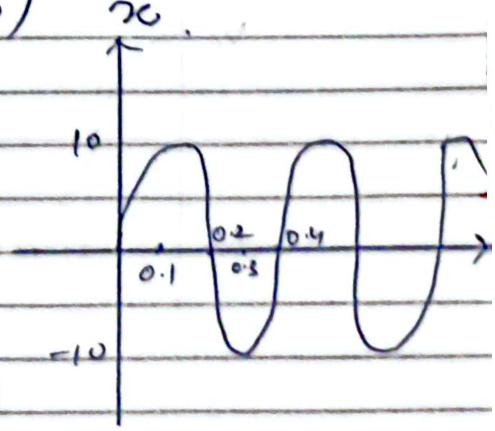
Write down equation for displacement

$$x = x_0 \sin \omega t$$

$$x = 10 \sin\left(\frac{2\pi}{T} t\right)$$

$$x = 10 \sin\left(\frac{2\pi}{0.4} t\right)$$

$$x = 10 \sin(5\pi t)$$



at	$t=0$	$x=0$
t	0.1	$x = 10$ cm
	0.2	$x=0$
	0.3	$x=-10$
	0.4	$x=0$

Equations for Velocity

$$x = x_0 \sin(\omega t)$$

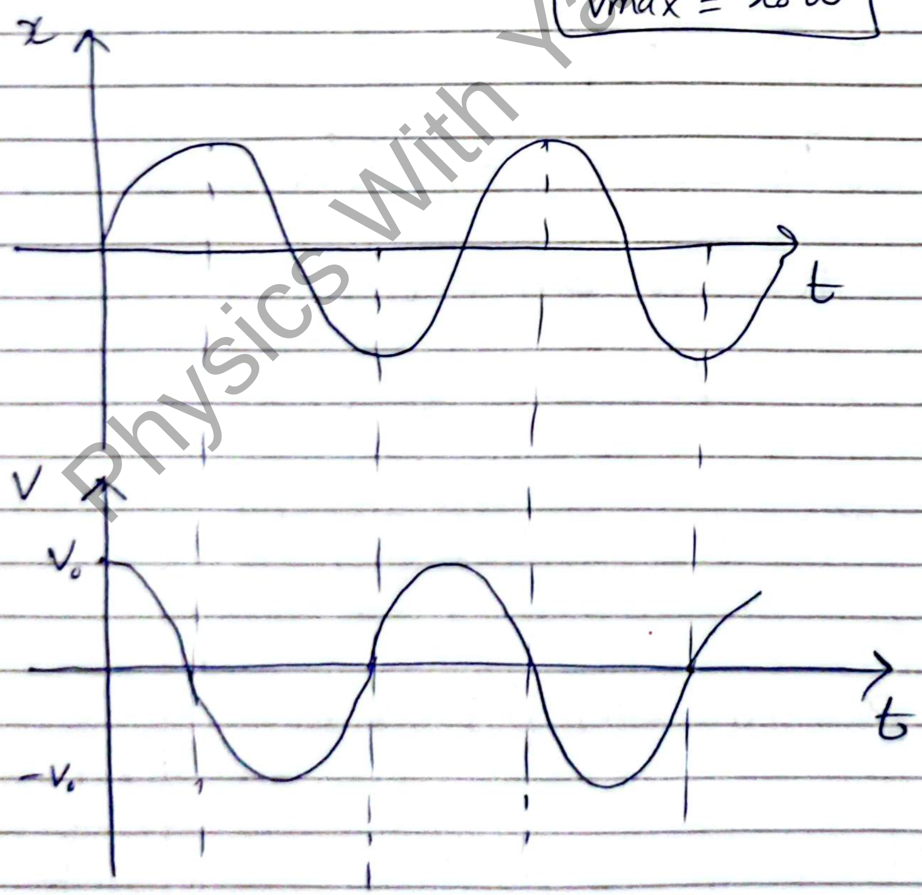
$$V = \frac{dx}{dt}$$

$$V = x_0 \cos(\omega t) \cdot \omega$$

$$V = x_0 \omega \cos(\omega t)$$

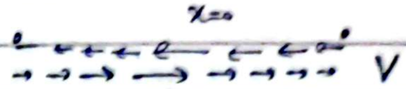
$$V = V_0 \cos(\omega t)$$

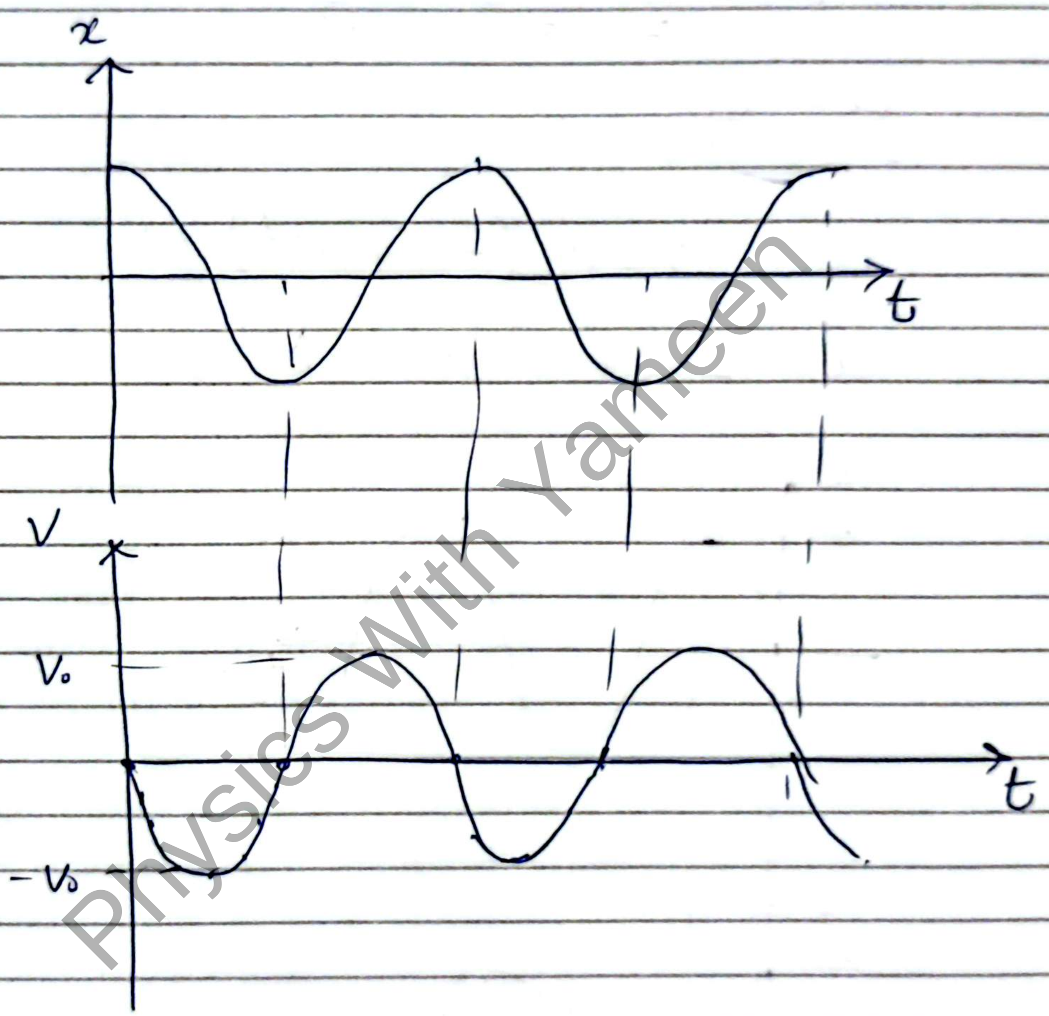
$$\boxed{V_{max} = x_0 \omega}$$



$x = x_0 \cos\omega t$

$V = -x_0\omega \sin\omega t$





Equations for acceleration

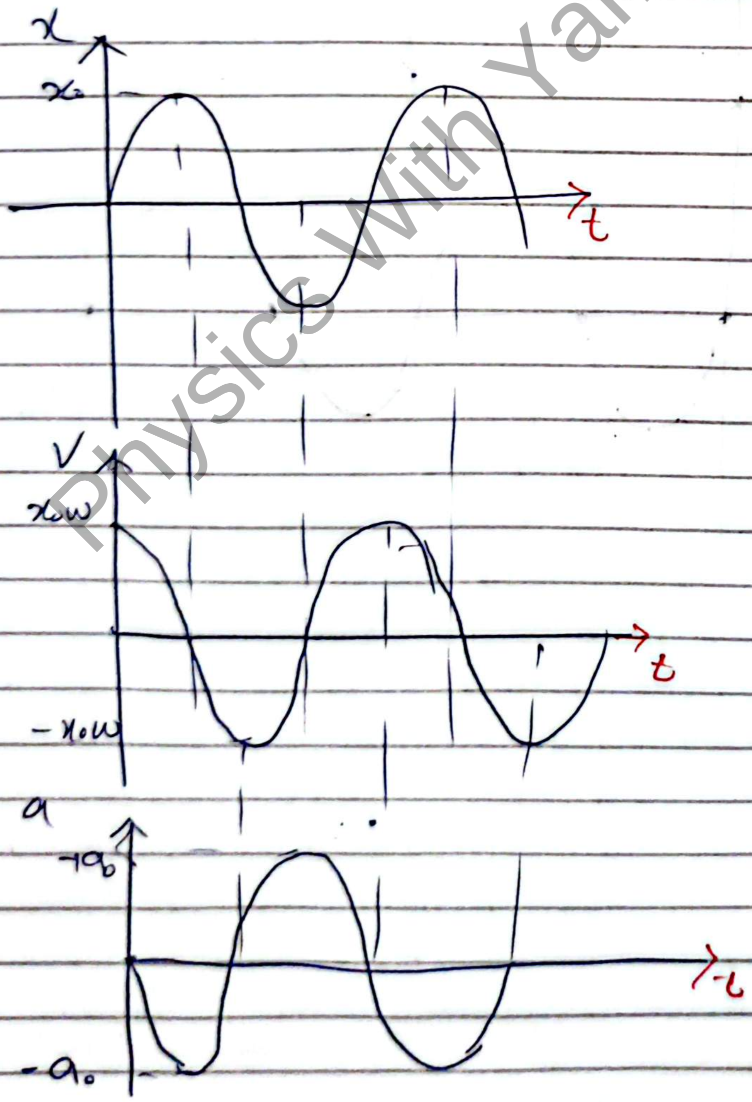
$$x = x_0 \sin \omega t$$

$$v = \underline{\underline{x_0 \omega}} \cos \omega \underline{\underline{t}}$$

$$a = \frac{dv}{dt}$$

$$a = x_0 \omega (-\sin \omega t) \cdot \omega$$

$$a = - x_0 \omega^2 \sin \omega t$$



Equation of velocity with respect displacement.

$$x = x_0 \sin\omega t \rightarrow (1)$$

$$V = x_0 \omega \cos\omega t \rightarrow (2)$$

from (1)

$$\frac{x}{x_0} = \sin(\omega t)$$

from (2)

$$\frac{V}{x_0\omega} = \cos(\omega t)$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\omega t + \cos^2\omega t = 1$$

$$\left(\frac{x}{x_0}\right)^2 + \left(\frac{V}{x_0\omega}\right)^2 = 1$$

$$\frac{x^2}{x_0^2} + \frac{V^2}{x_0^2\omega^2} = 1$$

$$\frac{\omega^2x^2 + V^2}{x_0^2\omega^2} = 1$$

$$\omega^2x^2 + V^2 = x_0^2\omega^2$$

$$V^2 = x_0^2\omega^2 - \omega^2x^2$$

$$V^2 = x_0^2\omega^2 - \omega^2 x^2$$

$$V = \pm\sqrt{x_0^2\omega^2 - \omega^2 x^2}$$

$$V = \pm\sqrt{\omega^2(x_0^2 - x^2)}$$

$$\boxed{V = \pm\,\omega\sqrt{x_0^2 - x^2}}$$

Derivation is not in the syllabus

at $x=0 \longrightarrow$ mean position

$$V = \pm\,\omega\sqrt{x_0^2 - 0^2}$$

$$\boxed{V_{max} = \pm\,\omega x_0}$$

at $x = x_0$ $\quad$ $V = 0$

↓

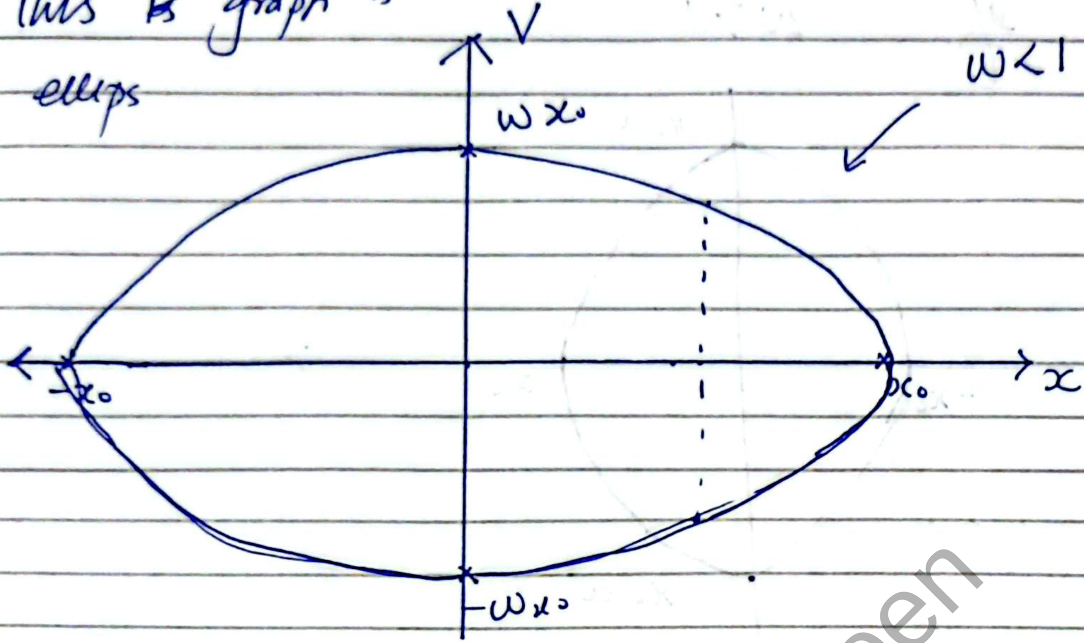
extreme point

If $x_0 = 4\,m$

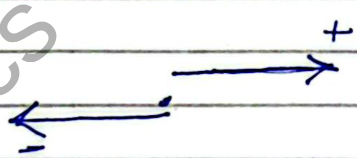
$\omega = 5$

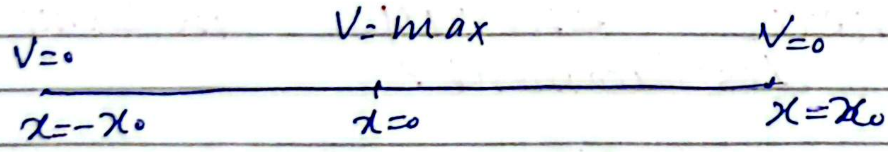
$$V = \pm 5\sqrt{16 - x^2}$$

This is graph 'is ellips

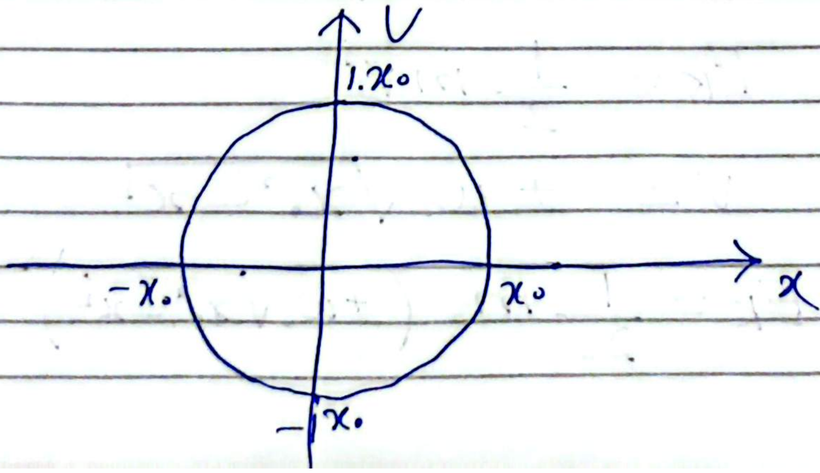


at any value of x there are two values of v one is positive and other is negative

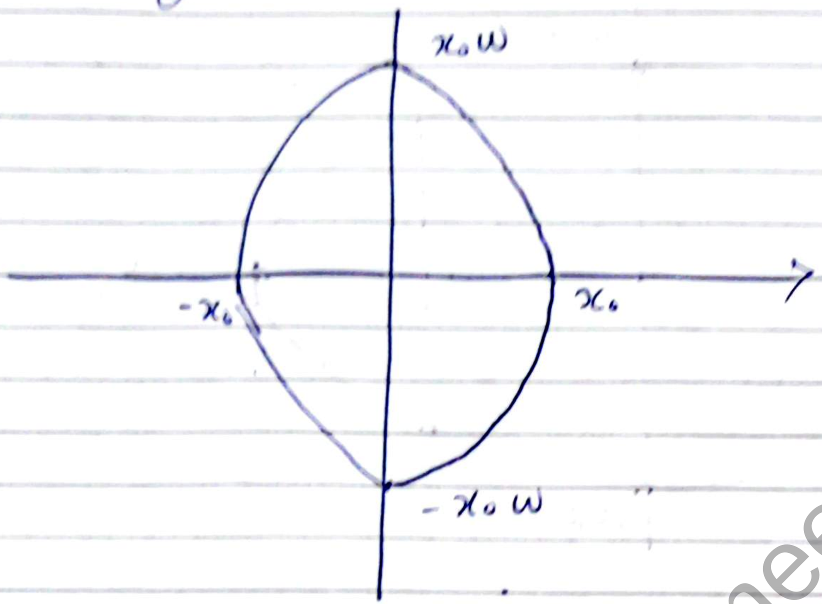




If $\omega = 1$



If $\omega > 1$



Kinetic energy in terms of displacement:

$$E_K = \frac{1}{2} m v^2$$

$$v = \pm \omega \sqrt{x_0^2 - x^2}$$

$$E_K = \frac{1}{2} m \left(\pm \omega \sqrt{x_0^2 - x^2} \right)^2$$

$$E_K = \frac{1}{2} m \omega^2 (x_0^2 - x^2)$$

at $x = 0$

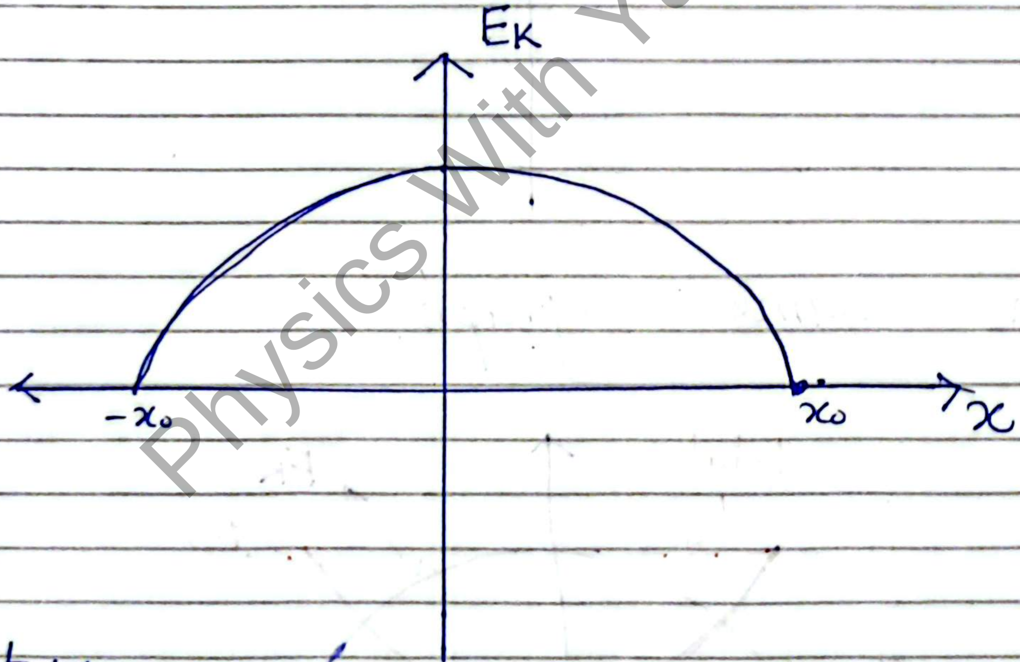
$$E_K = \frac{1}{2} m \omega^2 x_0^2$$ This is max

This is total energy

at $x = x_0$

$$E_K = \frac{1}{2} m \omega^2 (x_0^2 - x_0^2)$$

$$\boxed{E_K = 0}$$



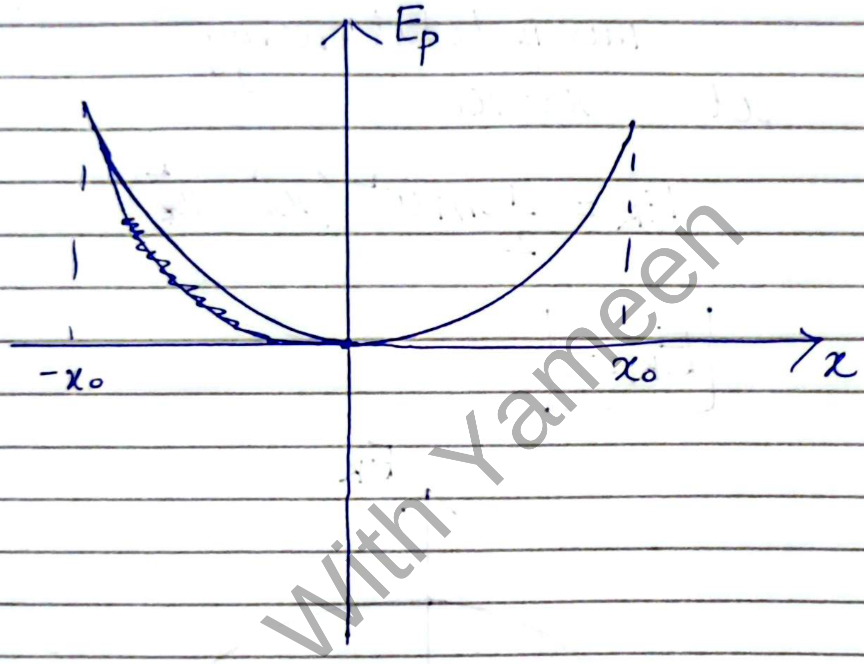
total energy: at mean K.E is max which is total energy $E_{Tot} = \frac{1}{2} m \omega^2 x_0^2$

$$E_p = E_{tot} - E_K$$

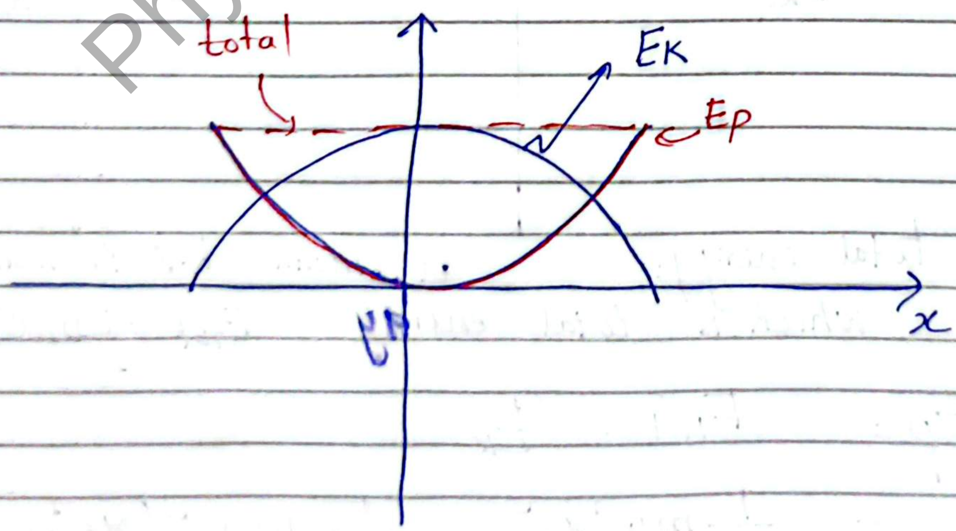
$$= \frac{1}{2} m \omega^2 x_0^2 - \frac{1}{2} m \omega^2 (x_0^2 - x^2)$$

$$E_p = \frac{1}{2}m\cancel{\omega^2 x_0^2} - \frac{1}{2}m\cancel{\omega^2 x_0^2} + \frac{1}{2}m\omega^2 x^2$$

$$\boxed{E_p = \frac{1}{2}m\omega^2 x^2}$$



on same graph



Energy variation with respect to time.

$$x = x_0 \sin\omega t$$

$$V = x_0 \omega \cos\omega t$$

$$E_K = \frac{1}{2} m v^2$$

$$E_K = \frac{1}{2} m (x_0 \omega \cos\omega t)^2$$

$$E_K = \frac{1}{2} m \omega^2 x_0^2 \underline{\cos^2\omega t}$$

at $t=0$ $\qquad \cos 0 = 1$

$$E_K = \frac{1}{2} m \omega^2 x_0^2$$

Potential energy

$$E_P = \frac{1}{2} m \omega^2 x^2$$

$$= \frac{1}{2} m \omega^2 (x_0 \sin\omega t)^2$$

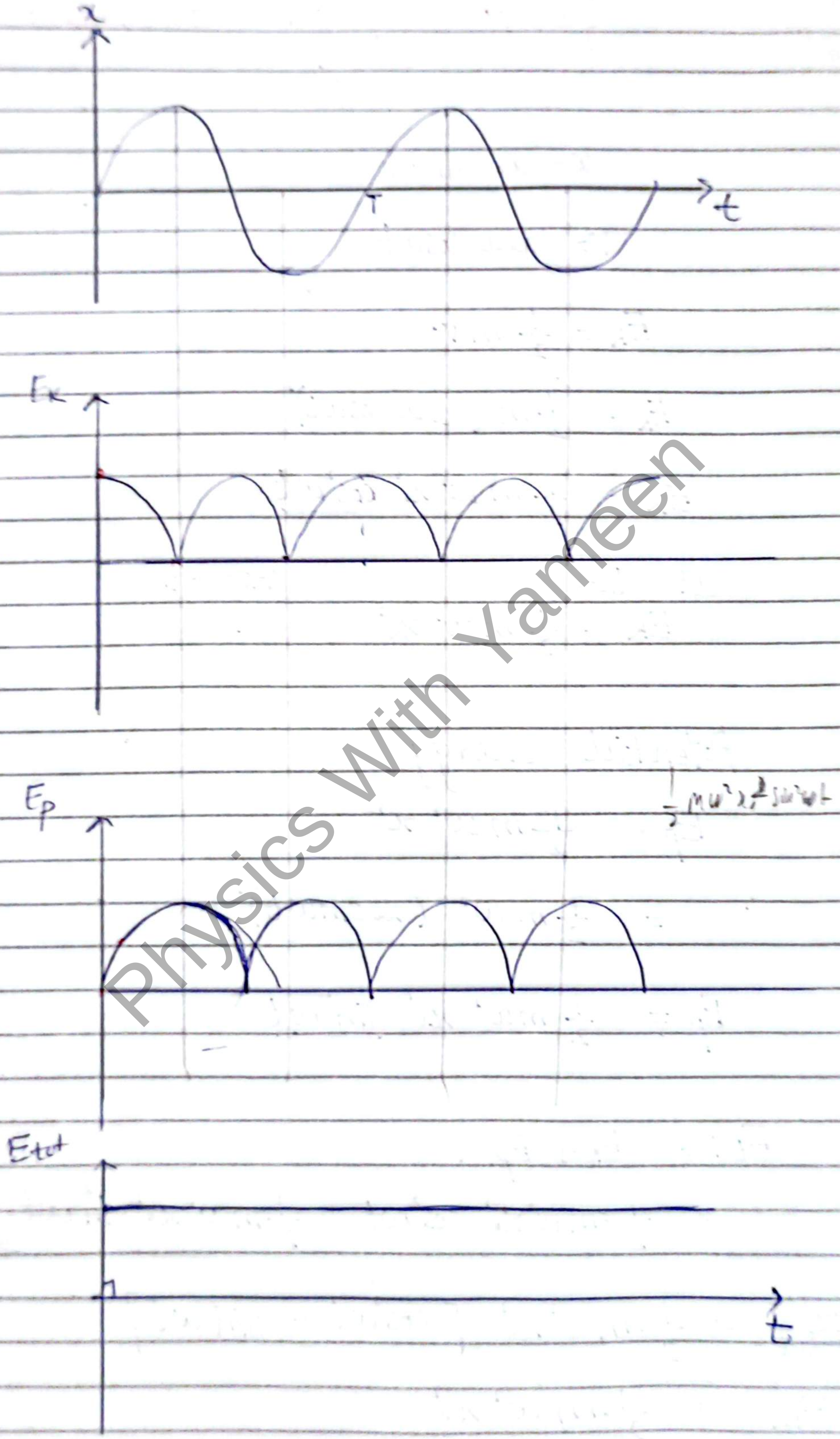
$$E_P = \frac{1}{2} m \omega^2 x_0^2 \underline{\sin^2\omega t}$$

$$E_{tot} = E_K + E_P$$

$$= \frac{1}{2} m \omega^2 x_0^2 \cos^2\omega t + \frac{1}{2} m \omega^2 x_0^2 \sin^2\omega t$$

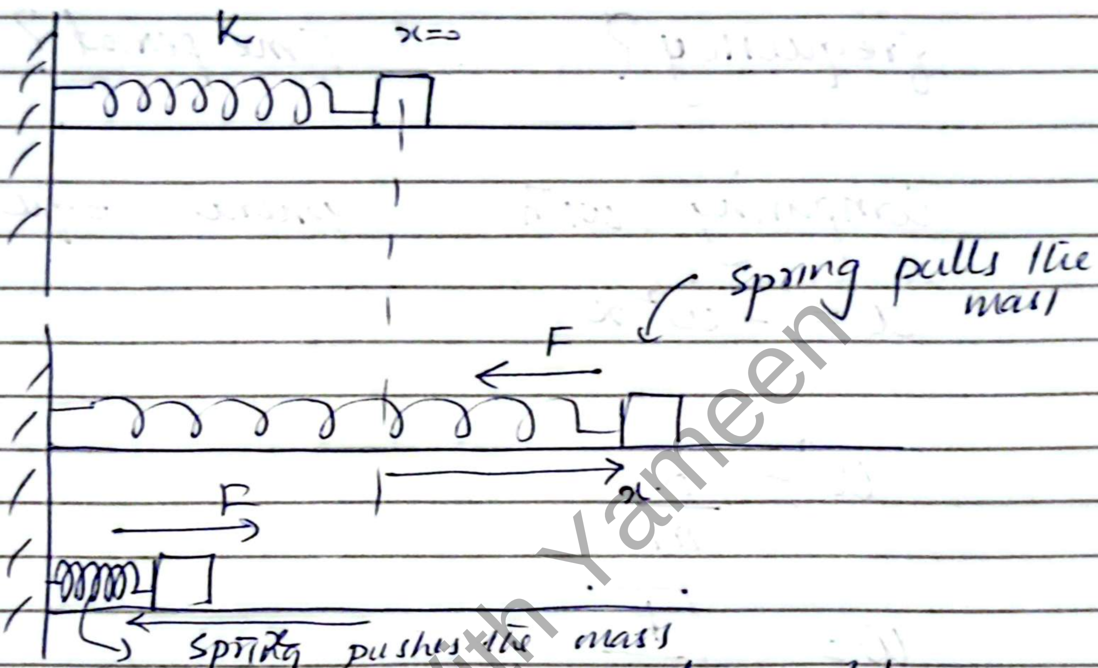
$$E_{tot} = \frac{1}{2} m \omega^2 x_0^2 [\cos^2\omega t + \sin^2\omega t]$$

$$E_{tot} = \frac{1}{2} m \omega^2 x_0^2$$



x
t
T
Ek
Ep
$\frac{1}{2} m\omega^2 x_0^2 \sin^2\omega t$
Etot
t

Examples of Simple Harmonic motion.



Mass is pulled towards right by some displacement x and then released

tension of the spring acts in backward direction

$$F = -kx \qquad \text{Hooke law}$$

$$ma = -kx$$

$$a = -\frac{k}{m}x$$

k and m are constant so

$a \propto x$ (−) shows a is towards mean

$$a = -\boxed{\frac{k}{m}}x$$

So it is simple Harmonic motion

frequency? Time period?

Comparing with general equation

$$a = -\boxed{\omega^2}x$$

$$\omega^2 = \frac{k}{m}$$

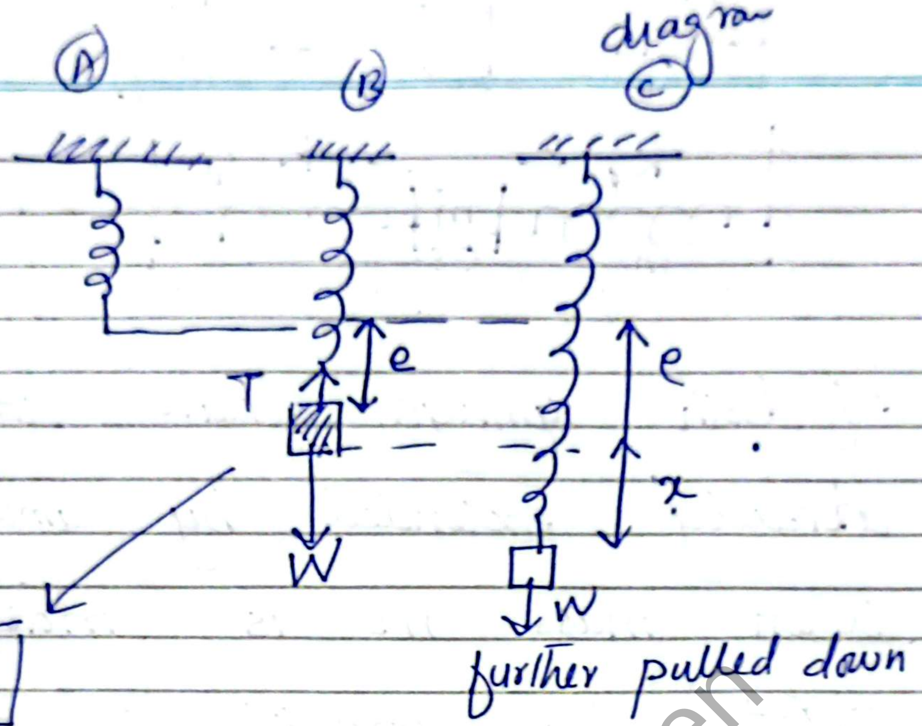
$$\omega = \sqrt{\frac{k}{m}}$$

$$T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{\sqrt{\frac{k}{m}}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

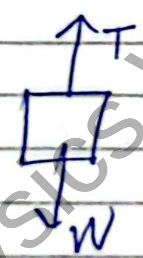
$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$



$\boxed{T = W}$

$ke = W$

In C Tension is greater than Weight



$\downarrow x\ (+)$

$F_{net} = -T + W$

$F = -k(x+e) + W$

$F_{net} = -kx - ke + W$

as $ke = W$

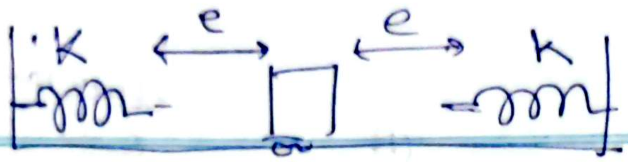
F_{net}

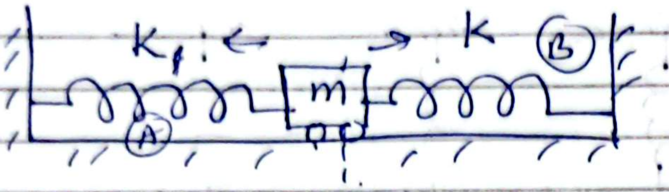
$F = -kx$

$ma = -kx$

$a = -\frac{k}{m}x$

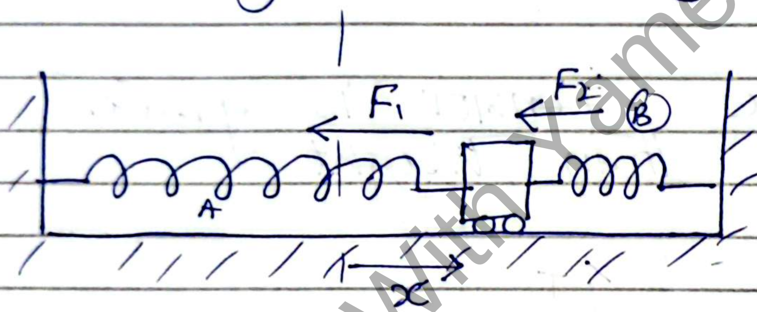
so SHM





If both springs have the already extended by amount 'e'

When mass m is moved towards right by amount of x



Spring A total extension

$$x+e$$

$$F_1 = k(x+e)$$

x is greater e

Spring B is compressed Now

total compression of spring B

$$x-e$$

$$F_2 = k(x-e)$$

(-) ←

→ x (+)

$-$: sign bcs of direction

$$F_{net} = -F_1 - F_2$$

$$F = -k(x+e) - k(x-e)$$

$$F = -kx - \cancel{ke} - kx + \cancel{ke}$$

$$F = -2kx$$

$$ma = -2kx$$

$$a = -\frac{2k}{m}x \rightarrow (1)$$

as k and m constant

so $a \propto x$

(-) sign shows a is towards equilibrium position

So obj performs S.H.M

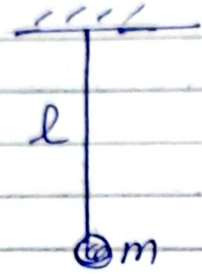
$$a = -\omega^2 x \rightarrow (2)$$

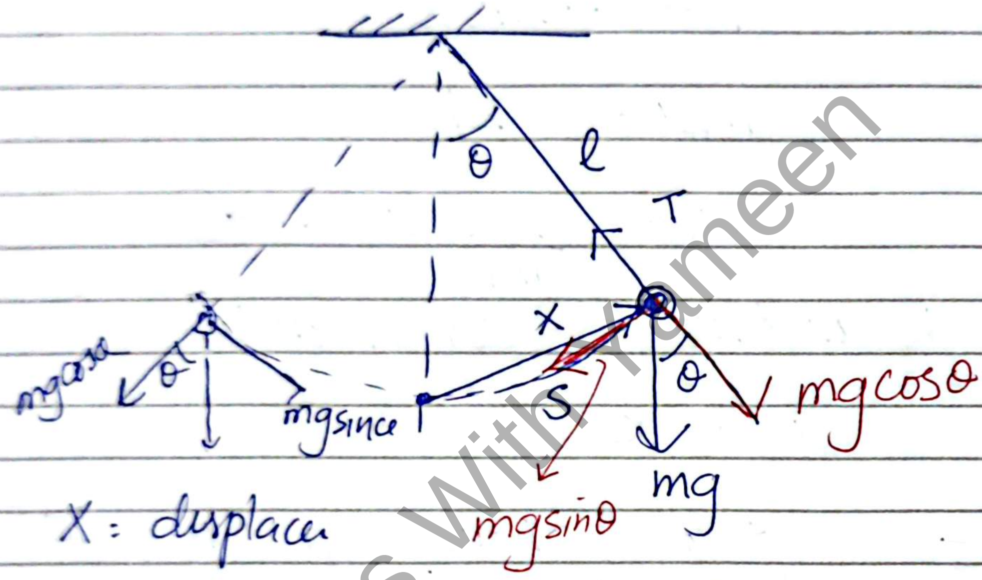
Comparing (1) and (2)

$$\omega^2 = \frac{2k}{m}$$

$$\omega = \sqrt{\frac{2k}{m}}$$

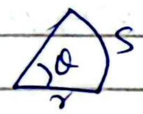
Simple pendulum





X: displace

$S = l\theta$



θ: radiam

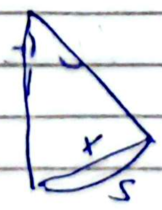
$F = mg \sin\theta$

when θ is small

$\sin\theta \approx \theta$: $\sin 0.1 = 0.099$

$x \approx S$

$F = mg\,\theta$



$\theta = \frac{x}{l}$

$$F = mg\frac{x}{l}.$$

$$F = \frac{mg}{l}x.$$

We can introduce (–) sign as x and F are in opposite direction

$$F = -\frac{mg}{l}x$$

$$ma = -\frac{mg}{l}x$$

$$a = \frac{-g}{l}x \longrightarrow (1)$$

g and l are constant

so $a \propto x$

Hence it performs SHM

ω? T? f?

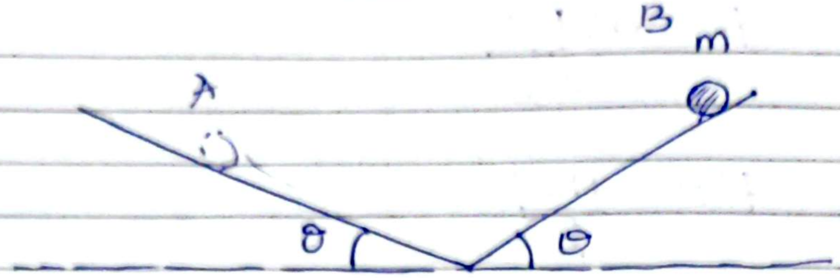
comparing with $a = -\omega^2 x \rightarrow (2)$

$$\omega^2 = \frac{g}{l} \qquad \omega = \sqrt{\frac{g}{l}}$$

$$T = \frac{2\pi}{\omega} =$$

$$T = \frac{2\pi}{\sqrt{g/l}} = 2\pi\sqrt{\frac{l}{g}}$$

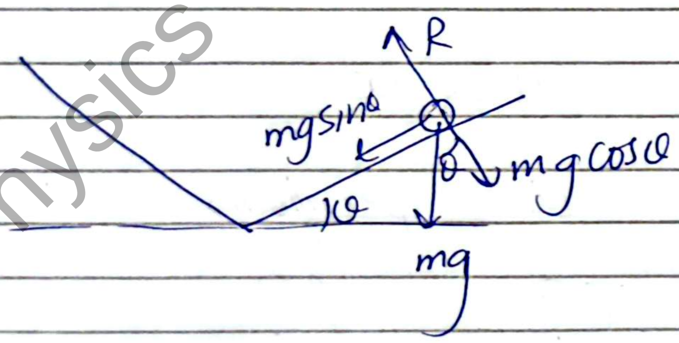
Example which is not SHM



an object of mass m is oscillating on inclined plane b/w A and B. Assume no friction.

Is this a simple Harmonic or not?

For this calculate acceleration



$F = mg\sin\theta$

θ: constant

$\cancel{m}a = \cancel{m}g\sin\theta$ $\quad a = g\sin\theta$

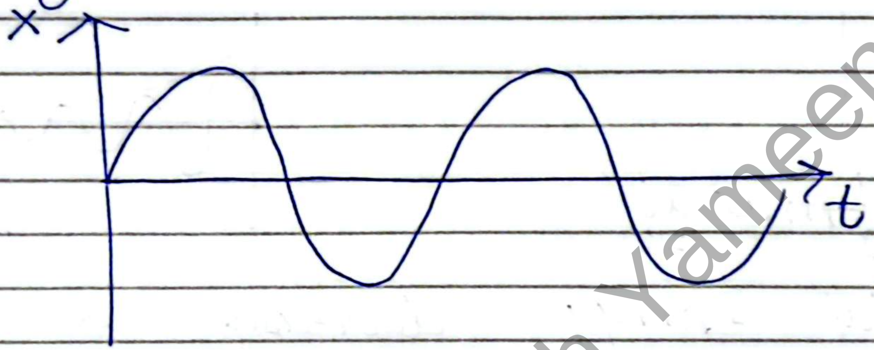
→ a is constant so not a S.H.M

Damping

Free oscillation

No loss of energy ~~due~~ or no gain of energy ~~to ex~~.

If there is no external resultant force.



amplitude should be constant

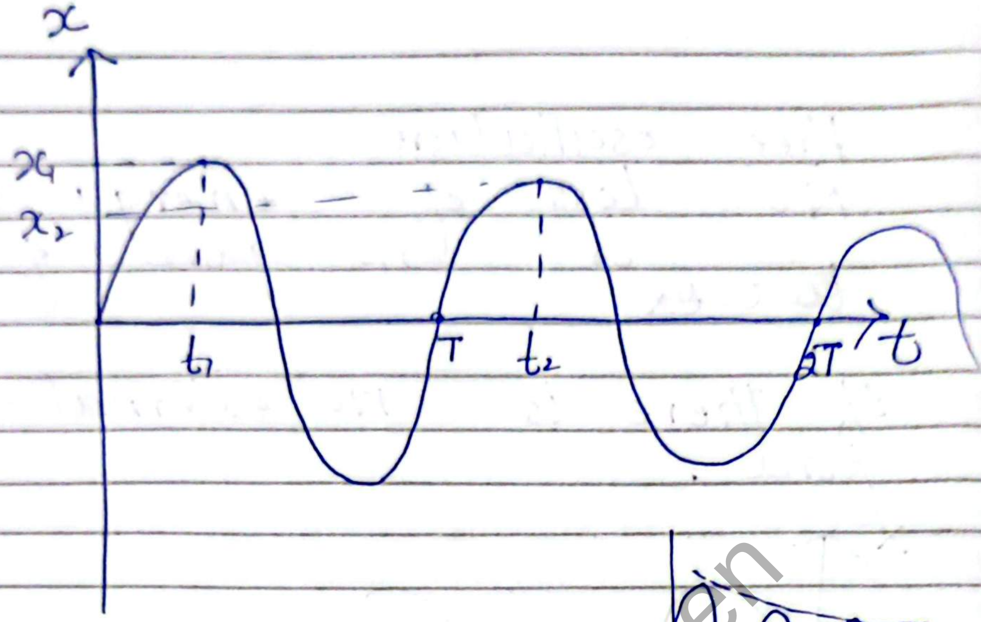
Damped oscillation

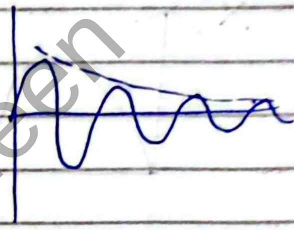
"Loss of energy due to external resistive forces"

Types

i) Light damping

oscillation is there but amplitude decreases slowly





$E_{tot} = \frac{1}{2} m \omega^2 x_0^2$

If total energy decreases so amplitude decreases.

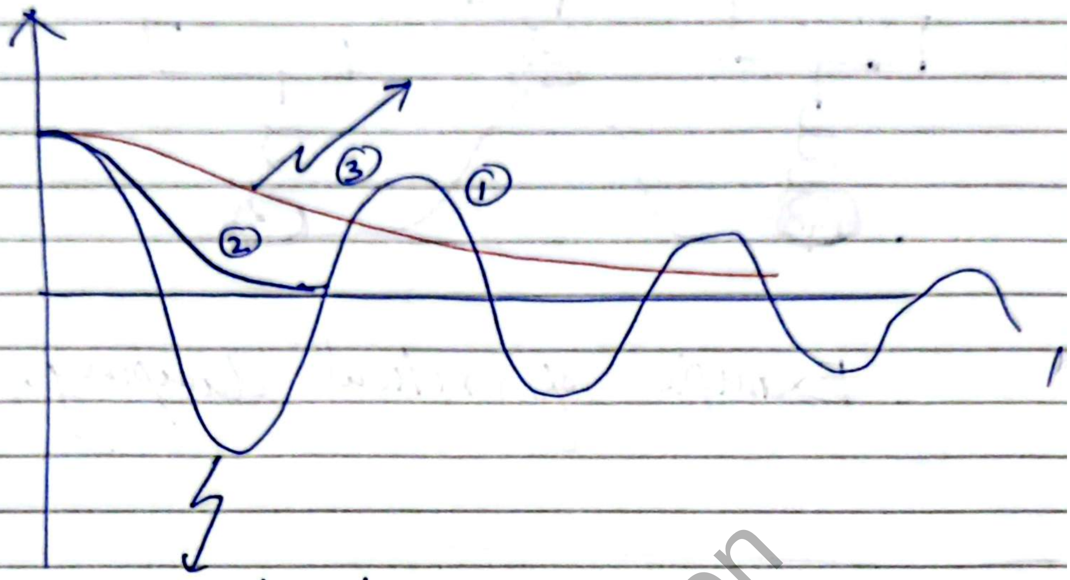
ω and T is constant.

Loss of energy from t_1 to t_2

$$E_{loss} = \frac{1}{2} m \omega^2 x_1^2 - \frac{1}{2} m \omega^2 x_2^2$$

$$= \frac{1}{2} m \omega^2 (x_1^2 - x_2^2)$$

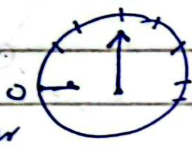
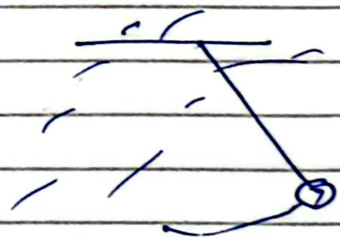
equation $x = x_0 \sin \omega t$ is not valid. why?



Light damping

2: Critical damping

(No oscillation

Example.  

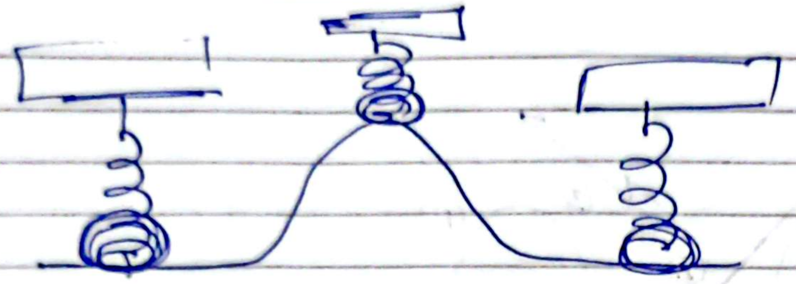
Shock absorber

3. over damping
heavy damping

* takes too much time to come to mean position

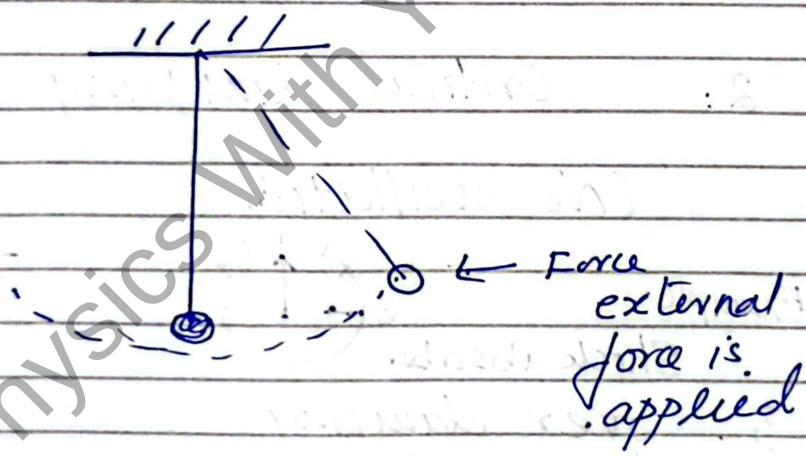
* No oscillation

Shock absorber



Example of critical damping.

Driven System

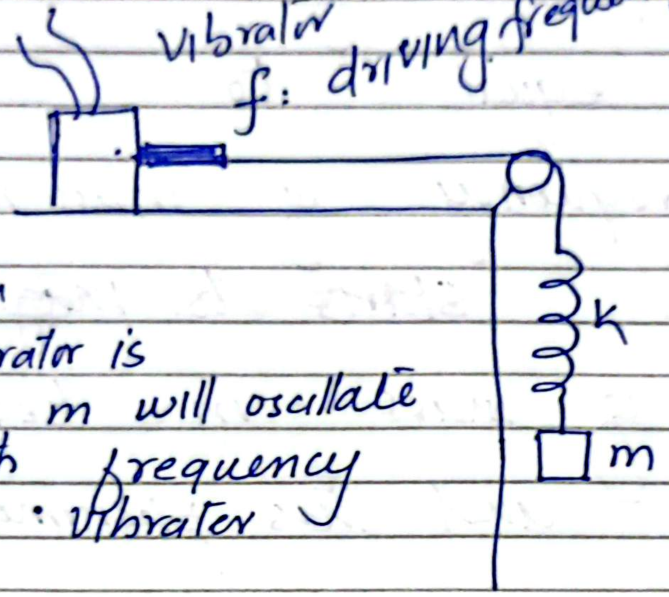


If energy is supplied (provided) to oscillating body externally by applying periodic force then it becomes driven system

AC saura

vibrator

f: driving frequency



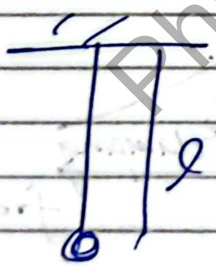
vibrater is off

when mass m is pulled down and released it will vibrate (oscillate) with its natural frequency

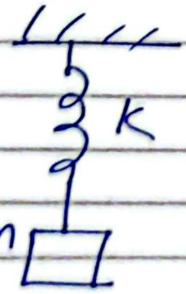
when vibrator is on m will oscillate with frequency of vibrater

Natural frequency

frequency with which an object will oscillate if no external net force is applied



$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$



$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

When frequency of vibrator is equal to natural frequency of mass m then the mass ~~starts to wag~~ amplitude of mass m becomes very large and this phenomenon is known as resonance.